A MODEL FOR THE STATISTICAL ANALYSIS OF LAND COMBAT SIMULATION AND FIELD EXPERIMENTATION DATA

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THESIS

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ABSTRACT

A nonstationary Markov model was developed relating weapon system effectiveness (WSE) to the time sequence of casualties observed in a two-sided, heterogeneous force, land combat simulation or field experiment. Parametric tests of hypothesis are used to analyse the relationships between WSE and engagement range and the numbers and types of combatants in the engagement. Maximum likelihood estimators of WSE demonstrated by a weapon system over the course of an engagement are obtained. Utilization of the Markovian model and WSE estimates in a low resolution simulation to investigate the impact of changes in force mix, speed of advance, and initial engagement range is discussed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF DRAWINGS

NOTATION

A: denotes force A with c combatant types.

A_i: denotes A combatant type i, i=1,...,c.

$A(s,\bar{m},\bar{n};t,k)$= the probability that the A combatants inflict k B casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$A_i(s,\bar{m},\bar{n};t,k)$= the probability that the A_i combatants inflict k B casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$A_{ij}(s,\bar{m},\bar{n};t,k)$= the probability that the A_i combatants inflict k B_j casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$a(t,\bar{m},\bar{n})=a\left[\bar{r}(t),\bar{m},\bar{n}\right]= \sum_{i=1}^{c} \sum_{j=1}^{d} a_{ij}(t,\bar{m},\bar{n}).$

$a_{ij}(t,\bar{m},\bar{n})=a_{ij}[\bar{r}(t),\bar{m},\bar{n}]$=the function used to define the probability of the A_i combatants inflicting B_j casualties given $\bar{m}$ and $\bar{n}$ survivers at time t, denoted as an attrition function.

B: denotes force B with d combatant types.

B_j: denotes B combatant type j, j=1,...,d.

$B(s,\bar{m},\bar{n};t,k)$= the probability that the B combatants inflict k A casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$B_j(s,\bar{m},\bar{n};t,k)$= the probability that the B_j combatants inflict k A casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$B_{ji}(s,\bar{m},\bar{n};t,k)$= the probability that the B_j combatants inflict k A_i casualties in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$b(t,\bar{m},\bar{n})=b[\bar{r}(t),\bar{m},\bar{n}]= \sum_{i=1}^{c} \sum_{j=1}^{d} b_{ji}(t,\bar{m},\bar{n}).$

$b_{ji}(t,\bar{m},\bar{n})=b_{ji}[\bar{r}(t),\bar{m},\bar{n}]$= the function used to define the probability of the B_j combatants inflicting A_i casualties given $\bar{m}$ and $\bar{n}$ survivers at time t, denoted as an attrition function.

K(l)= the number of casualties observed in the lth engagement of a simulation or field experiment.

L= the number of engagements or trails of a simulation or field experiment observed.

$L(\bar{p})$= the likelihood function for an observed sequence of casualties and times between casualties with the set $\bar{p}$ of unknown parameters.

$\bar{M}(t)$= a cX1 random vector $[M_i(t)]$, the number of A survivers at time t, with realization $\bar{m}$.

$M_i(t)$= a random variable, the number of A_i survivers at time t, i=1,...,c, with realization m_i.

$\bar{m}_{kl}$= the cX1 vector of A survivers after the k-1st casualty and prior to the kth casualty in the lth engagement.

$\bar{N}(t)$= a dX1 random vector $[N_j(t)]$, the number of B survivers at time t, with realization $\bar{n}$.

$N_j(t)$= a random variable, the number of B_j survivers at time t, j=1,...,d, with realization n_j.

$\bar{n}_{kl}$= the dX1 vector of B survivers after the k-1st casualty and prior to the kth casualty in the lth engagement.

$P(s,\bar{m},\bar{n};t,k)$= the probability that the A and B combatants inflict a total of k casualties on each other in the time interval (s,t], given $\bar{m}$ and $\bar{n}$ survivers at time s.

$\bar{p}$= the set of unknown attrition function parameters.

$\bar{r}(t)$= the matrix of ranges between opposing combatants at time t.

$\bar{r}_l(t)$= the matrix of ranges between opposing combatants at time t in the lth engagement.

t_{0l}= the time the lth engagement commences.

t_{kl}= the time of occurrence of the kth casualty in the lth engagement.

t_{el}= the time the lth engagement ends.

$$x_{ijkl} = \begin{cases} 1, \text{ if the kth casualty in the lth engagement was a } B_j \text{ combatant} \\ \text{and was inflicted by an } A_i \text{ combatant,} \\ 0, \text{ otherwise, } i=1,\ldots,c \text{ and } j=1,\ldots,d. \end{cases}$$

$$y_{jikl} = \begin{cases} 1, \text{ if the kth casualty in the lth engagement was an } A_i \text{ combatant} \\ \text{and was inflicted by a } B_j \text{ combatant,} \\ 0, \text{ otherwise, } i=1,\ldots,c \text{ and } j=1,\ldots,d. \end{cases}$$

I. INTRODUCTION

A. GENERAL

The purpose of this paper was to develop a model and consequent procedures for the statistical analysis of land combat simulation and field experimentation data. The model was the result of an effort to provide a framework for the statistical analysis of data obtained from two-sided, real-time-casualty-assessment field experiments conducted by the U.S. Army Combat Developments Experimentation Command (CDEC). Typically these experiments chiefly addressed the measurement of weapon system effectiveness (WSE) and, specifically, the contribution of particular weapon systems to total force effectiveness under a given scenario. In these experiments, WSE was measured in terms of the capability of a given weapon system or combatant type, rather than the capability of each individual combatant.

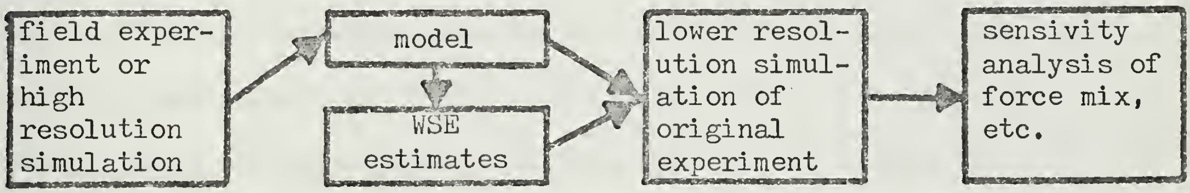
The principal direction of this paper was the derivation and estimation of measures of WSE, recognizing that WSE is the result of interactions among a number of factors describing the physical state of the engagement and the actions and decisions of the men employing the weapon systems. These factors were categorized as: the physical or performance characteristics of each combatant type, the numbers and types of combatants in the engagement, the engagement ranges, and other factors such as morale, terrain, and tactics. WSE is commonly assumed to be related to weapon performance characteristics such as rate of fire and probability of a hit. Often denoted as measures of effectiveness, such factors may be experimentally determined, but their relation to WSE is not always clear. The capability of a given combatant type would also

be expected to depend on the number of weapons of that type engaged in the battle. Additionally, WSE may involve interactions with other weapon types and depend on the numbers and types of supporting and opposing weapons. Further, WSE may vary with engagement range through the dependance of certain weapon performance characteristics on range. Finally, WSE is related to various other factors, such as morale and tactics, which are difficult to quantify and are generally fixed by the simulation or field experiment scenario.

Generally, loss exchange ratios (LER) have been the principal measure of WSE used in the analysis of the CDEC field experiments. LER are usually defined as the ratio of enemy to friendly losses over the course of a battle or a given time period. LER are time average measures of relative WSE and, hence, are sensitive to the duration of the engagement and the sequence of casualties experienced. Moreover, LER treat the relations between WSE and the physical state of the engagement only implicitly. Hence, LER may mask the possible dependance of WSE on such factors as force mixes, opening range of an engagement, or rate of advance.

This study proposes quantifying WSE in terms of a weapon's capability or potential for inflicting casualties. The proposed estimate of WSE explicitly evaluates the dependance of weapon system capability on range and the types and numbers of friendly and enemy combatants in the engagement as well as implicitly considering the relation between WSE and weapon performance characteristics. The estimate is an absolute measure of WSE as opposed to a relative measure as attained by LER. The WSE estimate complements rather than supplements the use of LER, as the former provides an instantaneous measure of capability while the latter is a time average measure.

In addition to providing WSE estimates, the structure of the proposed model and the resulting estimates of WSE may be used to construct a lower-resolution, computer simulation of the engagement using minimal computer resources. The simulation may be used to investigate, within the general limitations of the initial scenario, the effects of varying such parameters as force mix, initial engagement range, and speed of advance on the course of the battle.



B. THE MODEL

The model is designed to investigate the attrition process in a two-sided engagement between heterogeneous forces. The attrition process is modeled as a nonstationary Markov process where the states of the process are defined as the numbers and types of surviving combatants and the ranges between opposing combatants. The transition probabilities between the Markov states depend upon the surviving combatants' capabilities or WSE which, in turn, depend on the performance characteristics of each weapon type, the numbers and types of surviving combatants, and the ranges between combatants. Thus, the model provides a means of relating the sequence of states and the times of state transitions observed during a battle to WSE. In turn, this allows for the estimation of WSE on the basis of the observed sequence of states and transition times. It is recognized that the course of the battle also depends on such factors as morale, terrain, and tactics; however, these factors are assumed constant over all replicates of the experiment and, as such, are implicitly included

in the transition probabilities. In this formulation, the estimate of WSE may be interpreted as the capability of a weapon system to inflict casualties and, specifically, as the rate at which a given weapon system inflicts casualties on a particular type of opposing weapon.

In considering the validity of Markov assumptions, the nature of the combat process must be considered. An essential feature of any combat situation is its dichotomy. It consists of both an evolving physical system of weapons and their environment and a developing set of plans and decisions of opposing commanders. Given a state of the physical system and a particular set of plans and decisions, it may be argued that the evolution of the engagement depends only on the interactions of two opposing physical systems and is statistically deterministic--at least to the time of the next human decision. Of course the human element of planning and decision making is anything but deterministic; under the same conditions, different commanders will often make different decisions. Thus a Markov model might prove a quite poor model of actual combat. However, the use of decision rules in computer simulations and, to a lesser degree, the use of a scenario in field experimentation nullifies this human variability. In the cases of simulation and field experimentation, the combat process has already been abstracted to a process with little or no significant human variability and whose future states depend only on the interactions of the opposing physical systems [Koopman 1970]. In addition, the Markov assumption implies that the future behavior of the engagement depends only on its current state, not on its past history. That is, if the course of the battle depends on the types and numbers of combatants, only the current numbers of survivers is important, not the sequence and times of casualties in reaching the current state.

The model is based on the standard assumptions for a multidimensional Markov death process: the independance of non-overlapping time intervals, the near zero probability of two or more casualties occurring simultaneously, and, for all firer-target combinations, the existence of probabilities of a given firer type inflicting a casualty on a given target type. Given these assumptions, it is possible to develop, as a function of WSE, Chapman-Kolomorgov equations for the probability of observing the occurrence of any particular vector of casualty types and numbers. However, the resulting difference-differential equations are susceptible to a general solution only in special cases [Clark 1969]. Fortunately, the general solution to the differential equation for the occurrence of zero casualties in a time interval is trivial. From this probability it is possible to obtain the general distribution of the time between casualties and formulate the likelihood function for an observed sequence of casualty types and intercasualty times. Consequently, maximum likelihood estimates of WSE or the rate at which a given weapon type inflicts casualties on a particular opposing weapon type may then be obtained from the likelihood function and simulation or field experimentation data.

C. WSE ESTIMATION

The quantification of WSE is a two-step procedure. Initially, the general nature of the functional relationship between WSE and the numbers and types of combatants and range must be determined. Secondly, given a general relation between WSE and the state of the system, the problem becomes one of estimating the unknown parameters of this relationship. For example, if it is determined that the effectiveness of a tank platoon against a given antitank system is proportional to the number of surviving

tanks, n, and increases linearly as range, r, decreases, the general form of the tanks' WSE would be: $WSE = n(a-br)$, where a and b are unknown parameters to be estimated.

The general form of the functional relationship of WSE to the state of the system is obtained via parametric tests of hypothesis based on the asymptotic distribution of the generalized likelihood ratio. Essentially, in this step various general forms of WSE are postulated and the form which most closely correlates with the data is selected. Note that this procedure results in a statistical model of the attrition process. The general form of the estimated WSE equation is not necessarily the actual relation between WSE and the state of the battle; however, it is the form which provides the best available predictor of the course of the battle. The use of asymptotic or large sample distributional properties of the generalized likelihood ratio is justified even when relatively few casualties of weapon type B, inflicted by weapon type A are observed since the analysis is also based on the observations of system states which led to casualties of another type or casualties inflicted by another weapon system type.

II. THE MODEL

A. GENERAL

A model was formulated to estimate weapon system effectiveness from the time sequence of casualties observed in a two-sided, heterogeneous force, land combat simulation or field experiment. It is implicitly assumed that each type of combatant has a quantifable capability to inflict casualties on each type of opposing combatant. This capability, which may be zero against some or all opposing combatant types, is denoted as an attrition function and is employed as a measure of WSE. The attrition function is assumed to be a function of weapon performance characteristics, the numbers and types of surviving combatants, and the ranges between combatants. The attrition function may reflect the values of other factors such as terrain, morale, and tactics, although these latter factors are assumed constant over all replicates of the engagement.

The engagement attrition process was modeled as a nonstationary Markov process where the states of the process are defined by the numbers and types of surviving combatants and the ranges between combatants. The transition probabilities between the Markov states were assumed to depend on the surviving combatants' capabilities or WSE, which, in turn, depend on the performance characteristics of each weapon type, the numbers and types of surviving combatants, and the ranges between combatants. With this formulation, a weapon system's attrition function or measure of WSE represents the rate at which the weapon system is capable of inflicting casualties on a particular type of opposing weapon.

The model is based on the standard assumptions for a multidimensional Markov death process: the independance of non-overlapping time intervals,

the near zero probability of two or more casualties occurring simultaneously, and the existence, for each firer-target combination, of probabilities of a given firer type inflicting a casualty on a particular target type. Given these assumptions, Chapman-Kolomorgov equations may be developed, as functions of WSE, for the probabilities of observing the occurrence of any particular vector of casualty types and numbers. The resulting difference-differential equations yield a general solution only in special cases, however, the general solution to the differential equation for the occurrence of zero casualties in a time interval is straightforward. This solution is then used in the development of estimators and tests of hypothesis for the statistical analysis of simulation and field experimentation data.

B. THE ENGAGEMENT

The two opposing forces are designated as force A and force B respectively. Force A has c combatant types with the ith combatant type designated as A_i, i=1,...,c. Force B has d combatant types with the jth combatant type designated as B_j, j=1,...,d. Combatant types are defined such that the separation between elements or individual combatants of a combatant type is negligible compared to the ranges to opposing combatant types. If the engagement scenario requires that identical types of combatants operate in widely separated groups, such groups are defined as distinct combatant types. A total of L independent replicates of the engagement are performed under the same scenario.

Data available from each replicate of the engagement consists of the initial force levels, the type and time of occurrence of each casualty, and the type of combatant inflicting each casualty. In addition, unless any range dependence of the attrition process can be ignored, the relation

of the ranges between combatants to time must be, at least approximately, known. This relationship need not be constant over the replicates of the experiment and may be based on a known functional relationship, the scenario, or experimental data.

C. ASSUMPTIONS

Define the random variables:

$M_i(t)$= The number of A_i survivors at time t, with realization m_i, i=1, ...,c, and

$N_j(t)$= The number of B_j survivors at time t, with realization n_j, j=1, ...,d.

Define the random vectors:

$\overline{M}(t)$= The cX1 vector $[M_i(t)]$ of A survivors at time t with realization $\overline{m}$, and

$\overline{N}(t)$= The dX1 vector $[N_i(t)]$ of B survivors at time t with realization $\overline{n}$.

Define the variables:

$r_{ij}(t)$= The range between combatants A_i and B_j at time t,

$\overline{r}(t)$= The matrix of ranges $[r_{ij}(t)]$ between opposing combatant types at time t,

r_{aij}= The maximum range at which combatant type A_i can engage combatant type B_j, and

r_{bij}= The maximum range at which combatant type B_i can engage combatant type A_j.

Define the probabilities:

$A_{ij}(s,\overline{m},\overline{n};t,k)$= The probability the A_i combatants inflict k B_j casualties in the time interval $(s,t]$, given $\overline{m}$ and $\overline{n}$ survivors at time s, and

$B_{ji}(s,\overline{m},\overline{n};t,k)$= The probability that the B_j combatants inflict k A_i casualties in the time interval $(s,t]$, given $\overline{m}$ and $\overline{n}$ survivers at time s.

Then the assumptions discussed above may be formally stated as:

1. Events in non-overlapping intervals of time are independent.

2. For a given number of surviving combatants, casualties occur independently of each other in the infinitesimal time interval $(t, t+h]$.

3. For a sufficiently small length of time h, there exist attrition functions $a_{ij}[\bar{r}(t),\bar{m},\bar{n}]$ and $b_{ji}[\bar{r}(t),\bar{m},\bar{n}]$, $i=1,\ldots,c$ and $j=1,\ldots,d$, such that the probabilities of A_i combatants inflicting k B_j casualties in the time interval $(t,t+h]$ are given by:

a. $A_{ij}(t,\bar{m},\bar{n};t+h,1)=a_{ij}[\bar{r}(t),\bar{m},\bar{n}]h+0(h)$,

b. $\sum_{k=2}^{n_j} A_{ij}(t,\bar{m},\bar{n};t+h,k)=0(h)$,

similarly the probabilities of B_j combatants inflicting k A_i casualties in the time interval (t,t+h) are given by:

c. $B_{ji}(t,\bar{m},\bar{n};t+h,1)=b_{ji}[\bar{r}(t),\bar{m},\bar{n}]h+0(h)$,

d. $\sum_{k=2}^{m_i} B_{ji}(t,\bar{m},\bar{n};t+h,k)=0(h)$,

with the boundary conditions:

$A_{ij}(t,\bar{m},\bar{n};t+h,0)=1$, if $m_i=0$, $n_j=0$, or $r_{ij}(t)>r_{aij}$;

$B_{ji}(t,\bar{m},\bar{n};t+h,0)=1$, if $m_i=0$, $n_j=0$, or $r_{ij}(t)>r_{bji}$;

$A_{ij}(t,\bar{m},\bar{n};t+h,k)=0$, if $k>n_j$; and

$B_{ji}(t,\bar{m},\bar{n};t+h,k)=0$, if $k>m_i$.

D. TRANSITION PROBABILITIES

Let $a_{ij}(t,\bar{m},\bar{n})=a_{ij}[\bar{r}(t),\bar{m},\bar{n}]$ and $b_{ji}(t,\bar{m},\bar{n})=b_{ji}[\bar{r}(t),\bar{m},\bar{n}]$. Define the probability:

$A_i(s,\bar{m},\bar{n};t,k)$= The probability that the A_i combatants inflict k B casualties in the time interval $(s,t]$, given $\bar{m}$ and $\bar{n}$ survivors at time s.

Then from the assumptions:

$$A_i(t,\bar{m},\bar{n};t+h,0)=\prod_{j=1}^{d} A_{ij}(t,\bar{m},\bar{n};t+h,0)=\prod_{j=1}^{d}\left[1-a_{ij}(t,\bar{m},\bar{n})h+0(h)\right]$$

$$=1-\left[\sum_{j=1}^{d} a_{ij}(t,\bar{m},\bar{n})\right]h+0(h),$$

$$A_i(t,\overline{m},\overline{n};t+h,1)=\sum_{j=1}^{d} A_{ij}(t,\overline{m},\overline{n};t+h,1)\prod_{\substack{k=1\\k\neq j}}^{d} A_{ik}(t,\overline{m},\overline{n};t+h,0)$$

$$=\sum_{j=1}^{d}\left[a_{ij}(t,\overline{m},\overline{n})h+0(h)\right]\prod_{\substack{k=1\\k\neq j}}^{d}\left[1-a_{ik}(t,\overline{m},\overline{n})h+0(h)\right]=\left[\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})\right]h+0(h),\text{ and}$$

$$\sum_{k=2}^{N} A_i(t,\overline{m},\overline{n};t+h,k)=0(h),\quad N=\sum_{j=1}^{d} n_j.$$

Analogous results follow for the attrition of A combatants by B_j combatants. Define the probability:

$A(s,\overline{m},\overline{n};t,k)$= The probability that the A combatants inflict k B casualties in the time interval $(s,t]$, given $\overline{m}$ and $\overline{n}$ survivors at time s.

Then from the equations above:

$$A(t,\overline{m},\overline{n};t+h,0)=\prod_{i=1}^{c} A_i(t,\overline{m},\overline{n};t+h,0)=\prod_{i=1}^{c}\left[1-\left[\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})\right]h+0(h)\right]$$

$$=1-\left[\sum_{i=1}^{c}\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})\right]h+0(h),$$

$$A(t,\overline{m},\overline{n};t+h,1)=\sum_{i=1}^{c} A_i(t,\overline{m},\overline{n};t+h,1)\prod_{\substack{k=1\\k\neq i}}^{c} A_k(t,\overline{m},\overline{n};t+h,0)$$

$$=\sum_{i=1}^{c}\left[\left[\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})\right]h+0(h)\right]\prod_{\substack{k=1\\k\neq i}}^{c}\left[1-\left[\sum_{j=1}^{d} a_{kj}(t,\overline{m},\overline{n})\right]h+0(h)\right]$$

$$=\left[\sum_{i=1}^{c}\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})\right]h+0(h),\text{ and}$$

$$\sum_{k=2}^{N} A(t,\overline{m},\overline{n};t+h,k)=0(h),\quad N=\sum_{j=1}^{d} n_j.$$

Again analogous results follow for the attrition of A combatants by B combatants.

E. CHAPMAN-KOLOMORGOV EQUATIONS

Define $a(t,\overline{m},\overline{n})=\sum_{i=1}^{c}\sum_{j=1}^{d} a_{ij}(t,\overline{m},\overline{n})$ and $b(t,\overline{m},\overline{n})=\sum_{i=1}^{c}\sum_{j=1}^{d} b_{ji}(t,\overline{m},\overline{n})$ and the probability:

$P(s,\overline{m},\overline{n};t,k)$= The probability A and B inflict a total of k casualties on each other in the time interval $(s,t]$, given $\overline{m}$ and $\overline{n}$ survivors at time s.

Then from the equations above:

$$P(t,\overline{m},\overline{n};t+h,0)=A(t,\overline{m},\overline{n};t+h,0)B(t,\overline{m},\overline{n};t+h,0)$$

$$=[1-a(t,\overline{m},\overline{n})h+0(h)]\,[1-b(t,\overline{m},\overline{n})h+0(h)]$$

$$=1-[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})]h+0(h),$$

$$P(t,\overline{m},\overline{n};t+h,1)=A(t,\overline{m},\overline{n};t+h,1)B(t,\overline{m},\overline{n};t+h,0)+A(t,\overline{m},\overline{n};t+h,0)B(t,\overline{m},\overline{n};t+h,1)$$

$$=[a(t,\overline{m},\overline{n})h+0(h)]\,[1-b(t,\overline{m},\overline{n})h+0(h)]+[1-a(t,\overline{m},\overline{n})h+0(h)]\,[b(t,\overline{m},\overline{n})h+0(h)]$$

$$=[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})]h+0(h), \text{ and}$$

$$\sum_{k=2}^{K}(t,\overline{m},\overline{n};t+h,k)=0(h),\quad K=\sum_{i=1}^{c} m_i+\sum_{j=1}^{d} n_j.$$

Then the Chapman-Kolomorgov equation for the occurrence of zero casualties in the time interval $(s,t+h]$, where h is an arbitrarily small length of time, is:

$$P(s,\overline{m},\overline{n};t+h,0)=P(s,\overline{m},\overline{n};t,0)P(t,\overline{m},\overline{n};t+h,0)$$

$$=P(s,\overline{m},\overline{n};t,0)[1-[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})]h+0(h)]$$

$$=P(s,\overline{m},\overline{n};t,0)-P(s,\overline{m},\overline{n};t,0)[[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})]h+0(h)],\quad t>s.$$

The Chapman-Kolomorgov equation yields the difference equation:

$$\frac{P(s,\overline{m},\overline{n};t+h,0)-P(s,\overline{m},\overline{n};t,0)}{h}=-P(s,\overline{m},\overline{n};t,0)[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})],$$

and, when h approaches zero, the differential equation:

$$\frac{d}{dt}P(s,\overline{m},\overline{n};t,0)=-[a(t,\overline{m},\overline{n})+b(t,\overline{m},\overline{n})]P(s,\overline{m},\overline{n};t,0).$$

Solving the above first order differential equation subject to the initial condition $P(s,\overline{m},\overline{n};s,0)=1$ results in:

$$P(s,\overline{m},\overline{n};t,0)=\exp[-\int_s^t[a(y,\overline{m},\overline{n})+b(y,\overline{m},\overline{n})]dy].$$

Similar differential equations may be obtained from the appropriate Chapman-Kolomorgov equations for the occurrence of any number of casualties in a time interval. However, the equations yield general results only with difficulty even for special forms of the attrition functions [Clark 1969]. Fortunately, the probability of observing no casualties in a time interval yields sufficient information for the statistical analysis of simulation and field experimentation data.

III. STATISTICAL ANALYSIS

A. GENERAL

The estimation of WSE or weapon system attrition functions from simulation and field experimentation data was based on the maximum likelihood criterion. The likelihood function for an observed sequence of casualties and intercasualty times was developed from the Markovian assumptions and Chapman-Kolomorgov equation by obtaining the distribution of the time between casualties and the conditional distribution of casualty type given the occurrence of a casualty. The estimation of WSE is then accomplished via a two-step procedure. Initially, the general nature of the functional relationship between a weapon system's attrition function and the numbers and types of combatants and the ranges between combatants must be determined. The form of the functional relationship of WSE to the state of the attrition process is selected by parametric tests of hypothesis based on the asymptotic distribution of the generalized likelihood ratio. Secondly, given the general form of the attrition function, the maximum likelihood criterion is used to estimate unknown parameters of the function.

B. INTERCASUALTY TIME

While the distribution of the number of survivors is relatively intractable, the distribution of the time between casualties can be obtained in a straightforward matter. Define the random variables:

T_i = The time of occurrence of the ith casualty, $i \geq 1$, with distribution function $F_i(t)$, density $f_i(t)$, and realization t_i; and

$C(s,t)$ = The number of casualties occurring in the time interval $(s,t]$.

Note that the event $(T_i > t)$ occurs if and only if the event $[C(t_{i-1},t)=0]$ occurs, hence $P(T_i > t)=P[C(t_{i-1},t)=0]$. If there are $\bar{m}$ and $\bar{n}$ survivors after the i-1st casualty, then

$$1-F_i(t)=P(T_i > t)=P[C(t_{i-1},t)=0]=P(t_{i-1},\bar{m},\bar{n};t,0)$$

$$=\exp[-\int_{t_{i-1}}^{t} [a(y,\bar{m},\bar{n})+b(y,\bar{m},\bar{n})]dy], \text{ and}$$

$$f_i(t)= -\frac{d}{dt}P(T_i > t)=[a(t,\bar{m},\bar{n})+b(t,\bar{m},\bar{n})]\exp[-\int_{t_{i-1}}^{t} [a(y,\bar{m},\bar{n})+b(y,\bar{m},\bar{n})]dy].$$

C. CASUALTY TYPE

If there are $\bar{m}$ and $\bar{n}$ survivors after the i-1st casualty and the ith casualty occurs at time t_i, then, from the assumptions, the probability that the ith casualty was of combatant type B_j and was inflicted by combatant type A_i is:

$$\frac{a_{ij}(t_i,\bar{m},\bar{n})h}{a(t_i,\bar{m},\bar{n})h+b(t_i,\bar{m},\bar{n})h} = \frac{a_{ij}(t_i,\bar{m},\bar{n})}{a(t_i,\bar{m},\bar{n})+b(t_i,\bar{m},\bar{n})}.$$

That is, given a casualty has occurred at time t_i, the random variable describing the type of the ith casualty and the type unit inflicting the ith casualty has a multinomial distribution.

D. MAXIMUM LIKELIHOOD ESTIMATORS

If there are $\bar{m}$ and $\bar{n}$ survivors after the i-1st casualty at time t_{i-1}, then the likelihood function associated with observing a B_j casualty inflicted by an A_i combatant at time t_i is:

$$\frac{a_{ij}(t_i,\bar{m},\bar{n})}{a(t_i,\bar{m},\bar{n})+b(t_i,\bar{m},\bar{n})}[a(t_i,\bar{m},\bar{n})+b(t_i,\bar{m},\bar{n})]\exp[-\int_{t_{i-1}}^{t_i} [a(y,\bar{m},\bar{n})+b(y,\bar{m},\bar{n})]dy]$$

$$=a_{ij}(t_i,\bar{m},\bar{n})\exp[-\int_{t_{i-1}}^{t_i} [a(y,\bar{m},\bar{n})+b(y,\bar{m},\bar{n})]dy].$$

To develop the likelihood function for an observed sequence of casualties and times between casualties, define the random variables and notation:

$$x_{ijkl} = \begin{cases} 1, & \text{if the kth casualty in the lth engagement was a } B_j \text{ combatant and was inflicted by an } A_i \text{ combatant,} \\ 0, & \text{otherwise, } i=1,\ldots,c \text{ and } j=1,\ldots,d, \end{cases}$$

$$y_{jikl} = \begin{cases} 1, & \text{if the kth casualty in the lth engagement was an } A_i \text{ combatant and was inflicted by a } B_j \text{ combatant,} \\ 0, & \text{otherwise, } i=1,\ldots,c \text{ and } j=1,\ldots,d, \end{cases}$$

$\bar{m}_{kl}$ = The cX1 vector of A survivors after the k-1st and prior to the kth casualty in the lth engagement,

$\bar{n}_{kl}$ = The dX1 vector of B survivors after the k-1st and prior to the kth casualty in the lth engagement,

t_{0l} = The time the lth engagement commences,

t_{kl} = The time of occurrence of the kth casualty in the lth engagement,

t_{el} = The time the lth engagement terminates,

L= The number of independent replicates of the engagement observed,

K(l)= The number of casualties observed in the lth engagement,

$\bar{r}_l(t)$= The matrix of ranges between combatant types at time t during engagement l, and

$\bar{p}$= The set of unknown attrition function parameters.

At this point it is convenient to write the attrition functions as $a_{ij}[\bar{r}_l(t),\bar{m},\bar{n}]$ and $b_{ji}[\bar{r}_l(t),\bar{m},\bar{n}]$ to reflect the possibility of changes in the relation between time and ranges over replicates of the engagement. If a sequence of casualties are observed over L independent replicates of the engagement, the associated likelihood function is given by:

$$\prod_{l=1}^{L} \prod_{k=1}^{K(l)} \prod_{i=1}^{c} \prod_{j=1}^{d} [a_{ij}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]]^{x_{ijkl}} [b_{ji}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]]^{y_{jikl}} X$$

$$\exp[-\int_{t_{(k-1)l}}^{t_{kl}} [a[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl}]+b[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl}]]dy]X$$

$$\exp[-\int_{t_{K(l)l}}^{t_{el}}[a[\bar{r}_l(y),\bar{m}_{el},\bar{n}_{el}]+b[\bar{r}_l(y),\bar{m}_{el},\bar{n}_{el}]]dy].$$

The last term above is contributed by the time from the K(l)th casualty to the end of each engagement. For convenience and recognizing the loss of some information, the latter term is dropped in the remaining discussion. With this simplification, the natural logarithm of the likelihood function is:

$$\ln L(\bar{p})=\sum_{l=1}^{L}\sum_{k=1}^{K(l)}\sum_{i=1}^{c}\sum_{j=1}^{d}[x_{ijkl}\ln[a_{ij}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]]+$$
$$y_{jikl}\ln[b_{ji}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]]]-\sum_{l=1}^{L}\sum_{k=1}^{K(l)}[\int_{t_{(k-1)l}}^{t_{kl}}[a[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl}]+$$
$$b[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl}]]dy] .$$

Taking the derivative of $\ln L(\bar{p})$ with respect to an unknown parameter z which appears only in the attrition function $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$ and setting the derivative equal to zero results in:

$$\frac{d\ln L(\bar{p})}{dz}=\frac{d\ln L(\bar{p})}{da_{pq}[\bar{r}(t),\bar{m},\bar{n}]}\times\frac{da_{pq}[\bar{r}(t),\bar{m},\bar{n}]}{dz}=\sum_{l=1}^{L}\sum_{k=1}^{K(l)}\frac{x_{pqkl}}{a_{pq}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]}\times$$
$$\frac{da_{pq}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl}]}{dz}-\sum_{l=1}^{L}\sum_{k=1}^{K(l)}\int_{t_{(k-1)l}}^{t_{kl}}\frac{da_{pq}[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl}]}{dz}dy=0,$$

a function of $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$. Hence obtaining maximum likelihood estimators of parameters of a single attrition function involves working only with the attrition function of interest and is independent of the form and parameters of other attrition functions.

Closed form solutions to the parameter estimators are attainable only in a few special cases and, in general, parameter estimation involves the solution of simultaneous nonlinear equations using numerical analysis techniques. However, the form of the likelihood equations readily lend

themselves to easy solution by simple analysis procedures. The parameter estimators for the various functional forms of the attrition function discussed in the appendices were found to rapidly converge to an apparently global solution using the basic numerical analysis technique of fixed point iteration [Conte 1972]. When an unknown parameter is common to several attrition functions, the derivative of ln $L(\overline{p})$ appears more formable, but does not greatly increase the difficulty of obtaining numerical solutions to parameter estimators.

Confidence intervals or regions about attrition function parameters may be established via the large sample distribution of maximum likelihood estimates. The maximum likelihood estimates $\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_k$ from a sample size n of the parameters of an attrition function $a(t,\overline{m},\overline{n})$ are approximately distributed as a k- variate normal with mean vector $(p_1, \ldots, p_k)$ and covariance matrix nR, where:

$$nR = \left(-nE\left[\frac{d^2}{dp_i dp_j} \ln a(t,m,n) \right] \right),$$

a kXk matrix evaluated at $p_i = \hat{p}_i$ [Rao 1965].

E. TESTS OF HYPOTHESIS

The general nature of the functional relationship between weapon systems' attrition functions and the numbers and types of combatants and ranges between combatants is determined by parametric tests of hypothesis. This step of the analysis is based on hypothesizing various potential general forms of the attrition functions and then conducting sequential tests of hypothesis to select the form which most closely agrees with the experimental data. The selection may be accomplished by simply hypothesizing a variety of functional forms and selecting from among them. Alternately, the selection process may start with a simple, say constant,

form of the attrition functions and sequentially test more and more complex forms until the addition of more parameters is no longer justified by improvements in the fit of the data. In any case, this procedure results in a correlation model of the attrition process, i.e., the form of the attrition function selected is not necessarily the actual relation between WSE and the state of the battle; however, it is the form which provides the best available predictor of the course of the battle.

The tests of hypothesis are based on the asymptotic distribution of the generalized likelihood ratio. Consider two hypothesized forms of the attrition functions with r and c unknown parameters respectively, $r > c$. If $L(\overline{p}_r)$ is the likelihood function for the first set of attrition functions evaluated at the maximum likelihood estimates of the r unknown parameters and $L(\overline{p}_c)$ is the likelihood function for the second set of attrition functions evaluated at the maximum likelihood estimates of the c unknown parameters, then $-2 \ln[L(\overline{p}_c)/L(\overline{p}_r)]$ asymptotically approaches a Chi-square distribution with r-c degrees of freedom [Rao 1965]. In the comparison of two hypotheses where the numbers of unknown parameters are equal, $r=c$, maximum likelihood criterion may be applied in the comparison of the two hypotheses. The hypothesis which results in the maximum value of the two likelihood functions evaluated at the maximum likelihood estimates of their parameters is selected. The use of asymptotic or large sample distributional properties of the generalized likelihood ratio is justified even when relatively few casualties of a given weapon type are observed since the test statistic is also based on the observations of system states which led to casualties of another type or casualties inflicted by another weapon system.

There are 2(cXd) attrition functions in the likelihood function and attempting to test composite hypotheses about this number of functions for the variety of permutations of possible functional forms would be time consuming at best. Fortunately, it is possible to reduce the test of hypothesis to a comparison of those attrition functions which have common parameters under one or both of the hypothesis. Consider the comparison of two hypotheses which differ in the form of the attrition function $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$ and where the other functions are assumed to have the same, though perhaps unknown, forms with no parameters common to $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$. If the associated likelihood functions are $L(\bar{p}_c)$ and $L(\bar{p}_r)$, containing c and r maximum likelihood estimates of unknown parameters of $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$ respectively, then:

$$-2\ln[L(\bar{p}_c)/L(\bar{p}_r)]=2\ln L(\bar{p}_r)-2\ln L(\bar{p}_c)=$$

$$2\sum_{l=1}^{L}\sum_{k=1}^{K(l)}[x_{pqkl}[\ln a_{pq}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl},r]-\ln a_{pq}[\bar{r}_l(t_{kl}),\bar{m}_{kl},\bar{n}_{kl},c]]-$$

$$\int_{t_{(k-1)l}}^{t_{kl}}[a_{pq}[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl},r]-a_{pq}[\bar{r}_l(y),\bar{m}_{kl},\bar{n}_{kl},c]]dy]\sim\chi^2_{r-c},$$

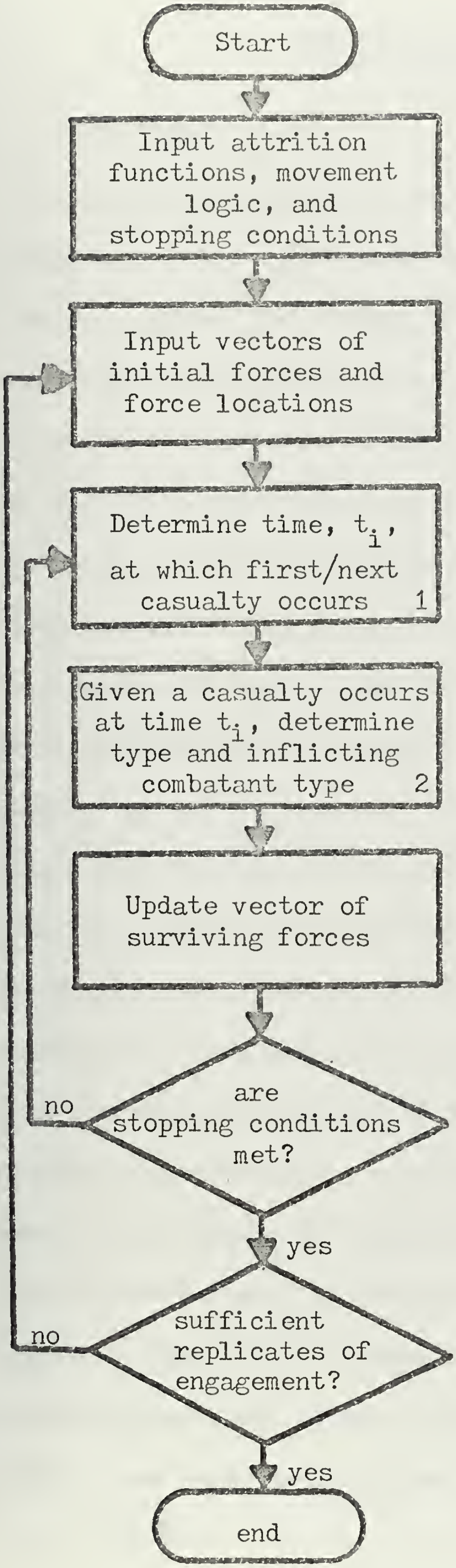
and the test statistic is a function of $a_{pq}[\bar{r}(t),\bar{m},\bar{n}]$ only. If several attrition functions share common parameters, then the forms of these functions must be compared against alternatives simultaneously and the hypothesis test statistic can be reduced only so far as an expression in these functions.

IV. MARKOV SIMULATION

Another consequence of the Markovian assumptions is the possibility of creating a computer simulation of the original simulation or field experiment using minimal computer resources. The Markov simulation is of lower resolution, but would be useful in investigating the effects of changes in those variables explicitly considered in the Markov model: the numbers and types of combatants, ranges, and the relationship between time and ranges. Potential applications include studies of the changes in engagement outcomes with changes in initial force mixes, initial engagement range, and speed of advance. As with all correlation models, caution must be exercised in considering parameter values outside the range of the data base; however, the simulation is intended more as a design tool for higher resolution experiments and should prove satisfactory in this role.

The simulation is based on the Markovian assumptions of the model and the attrition functions estimated from simulation or field experimentation data. An implication of the Markov model is that the battle may be regarded as a sequence of observations of two random variables: the time to the next casualty given a particular force level and the type of casualty observed given a casualty has occurred. The simulation simply involves specifying the initial forces, estimated attrition functions, relationship between time and ranges, and the engagement stopping conditions and then, using monte carlo procedures, generating a sequence of observations of the two random variables discussed above. A flow chart of such a simulation is indicated in Figure 1.

Markov Simulation Flow Chart



[1] The distribution function of the time to the ith casualty is given by:

$$P(T_i > t) = \exp\left[-\int_{t_{i-1}}^{t} [\hat{a}(y,\bar{m},\bar{n})+\hat{b}(y,\bar{m},\bar{n})]dy\right].$$

[2] The random variable describing the type and inflicting unit of the ith casualty, given a casualty has occurred at time t_i and $\bar{m}_i$ and $\bar{n}_i$ survivors prior to the casualty, has a multinomial distribution. The probability the casualty was of type B_j and was inflicted by an A_i combatant is given by:

$$\hat{a}_{ij}(t_i,\bar{m}_i,\bar{n}_i)/[\hat{a}(t_i,\bar{m}_i,\bar{n}_i)+\hat{b}(t_i,\bar{m}_i,\bar{n}_i)].$$

Figure 1

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The Markov model yields an estimate of a weapon system's effectiveness measured in terms of the rate at which the weapon system inflicts casualties on a given type of opposing weapon. The estimate is not a single measurement as are LER, but an evaluation of a continuous function which indicates the variation of WSE over the course of the battle as ranges and the types and numbers of combatants change. It is essential to differentiate between the estimated attrition functions and the values of these functions when evaluated over the observed values of $\overline{r}(t)$, $\overline{m}$, and $\overline{n}$. The analysis procedure guarantees only that it will select the variables and functional relationships which produce values of WSE which most closely agree with observed time sequence of casualties, not that it will select the attrition function which most closely agrees with the relationship between WSE and the state of the attrition process. Essentially, the Markov model converts the information contained in an observed time sequence of casualties into a continuous function which, when evaluated over the observed sequence of casualties, provides a measure of the capability demonstrated by a weapon system over the course of the engagement. For example, an analysis of 31 trials from CDEC experiment 43.6 yielded the estimate of WSE for TOW equipped COBRA helicopters against aggressor tanks shown in Figure 2A. Effectiveness, measured as aggressor tanks destroyed per minute, is plotted against the range between the COBRA's and tanks and represents the average effectiveness observed over the 31 trials.

The estimates of WSE for individual weapon types may be combined to obtain the total effectiveness of a force against a given opposing weapon since, under the Markovian assumptions, the estimates are simply additative. Given the validity of these assumptions, adding WSE does not inflate the estimate of total force capability as the estimation procedure for individual weapon type WSE is based on this additative property. Figure 2B, again based on CDEC experiment 43.6 data, plots the average observed total effectiveness of the defending force against the aggressor tanks. The plot also shows the contribution of the COBRA helicopters to the total force effectiveness.

It should be emphasized again that the procedure discussed above does not predict WSE, but transforms information on WSE contained in an observed time sequence of casualties into a single dimension, a series of numbers which, in some sense, measure WSE and indicate how it varied over the course of a battle. An intermediate step in such an analysis is the development of a function which relates WSE to ranges and the numbers and types of combatants. Caution should be exercised in the use of this function for predictive purposes as the Markov model guarantees only a correlation model of the observed engagement. For example, the function giving the effectiveness of the COBRA helicopters against aggressor tanks discussed above indicates that WSE is proportional to the number of target tanks and increases linearly with decreasing range. As far as it goes, such a relationship is reasonable; however, the function suggests the COBRA force WSE is invarient as the number of COBRA's in the force changes. While possibly true, a more likely explanation is the fact that the data base used included very few COBRA losses and, to the Markov model, the number of COBRA's was essentially constant and, hence, not an important

COBRA Capability Against Aggressor Tanks vs Range

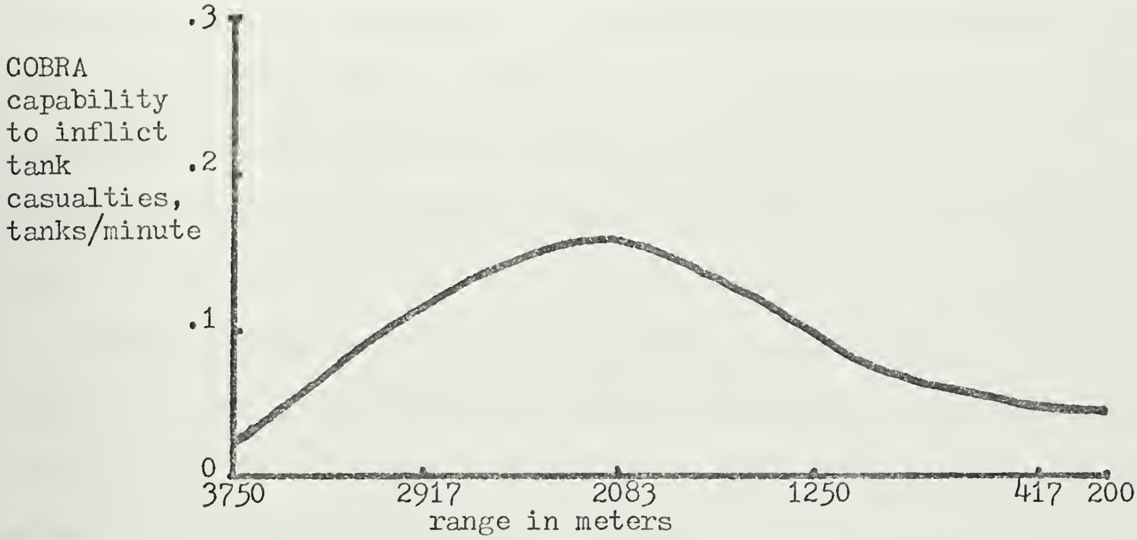
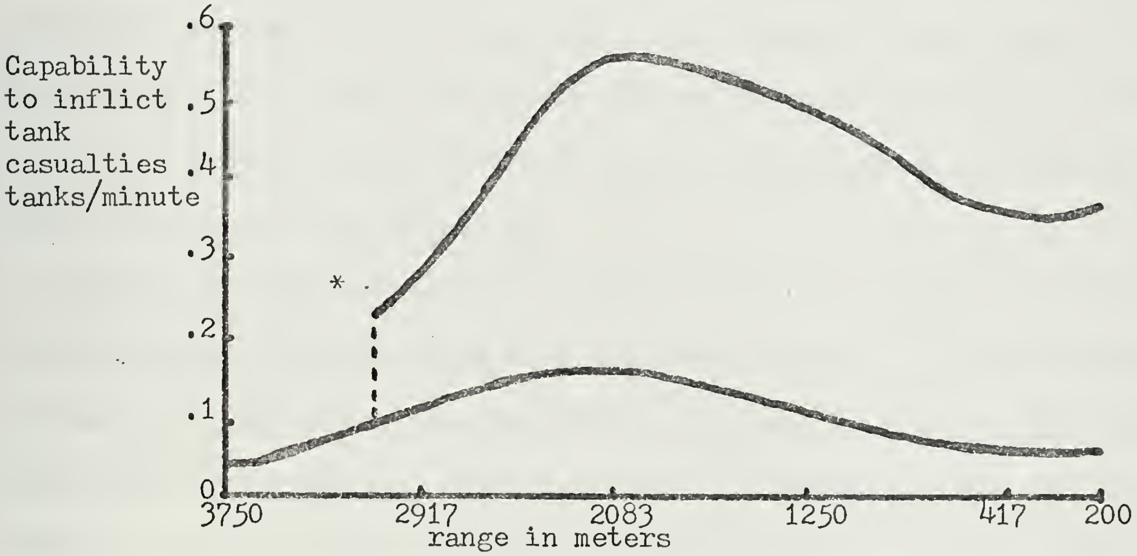


Figure 2A

Comparison of Total Force and COBRA Capability Against Aggressor Tanks



* Only COBRA able to acquire targets beyond this range

Figure 2B

variable in predicting the WSE of the COBRA force. Thus, using this function to predict the effectiveness of a different number of COBRA helicopters against tanks might be quite misleading; nevertheless, the estimates of WSE values for COBRA's obtained from the data are perfectly acceptable. This is simply an example of the risk, present in all correlation models, of attempting a prediction beyond the range of the data base.

Another limitation of the Markov model, even when within or near the range of the data base, is the fact that the estimate of WSE effectiveness is a function of two random vectors, the vectors of surviving combatants, whose distribution is, in general, unknown. It is possible to make qualitative predictions about the effects on engagement outcomes of changes in initial force mixes, speed of advance, and the like. However, quantitative answers to such questions as expected losses or casualties inflicted or the probability of winning can be addressed only by use of the Markov simulation. While the Markov simulation requires relatively little in the way of computer resources, its use is less desirable than an analytic solution to these questions.

Overall, the Markov model is primarily useful in quantifying WSE observed in simulation and field experimentation trials. If the limitations on its applicability are recognized, the model may be used to make predictions about the effects of scenario changes on engagement outcomes. Appendices A and B contain examples of applications of the Markov model to the analysis of simulation and field experimentation data respectively.

B. RECOMMENDATIONS

Further work is needed in the validation of the assumptions used in the Markov model; however, given acceptance of these assumptions, the

model is ideally suited to the development of a standardized computer package for the quantification of WSE in the analysis of simulations and field experiments yielding time histories of casualty occurrence. Such an analysis package would use sequential tests of hypothesis to select from potential general forms of the attrition function and would develop maximum likelihood estimates of unknown parameters. The use of sequential tests of hypothesis requires that the variety of potential general forms of the attrition functions be reduced to a computationally manageable number, but remain comprehensive enough to include the variety of relationships between WSE and the state of the attrition process which experience indicates may exist. Appendix C includes an attempt to develop such a selection. The analysis capability should be complemented by the capability of using the estimated attrition functions and casualty-time data to quantify and display the weapon capabilities indicated by the data. A crude version of such a package was constructed, using the attrition function forms in Appendix C, for the analyses presented in Appendices A and B. A similar standardized program could be developed for the Markov simulation for such applications as experimental design of higher resolution simulations and field experiments.

Appendix A. Analysis of Computer Simulation Data

1. General

This appendix presents the results of an analysis of computer simulation data using the Markov model. The data was generated using the Markov simulation outlined above in order to exercise the analysis procedure in a situation where the attrition functions were known and the Markov assumptions valid.

2. Simulation

The simulation represented a hypothetical engagement between a stationary, defending A force and an attacking B force. The A force was composed of two combatant types: A_1 and A_2, each with two elements. The B force was composed of two identical combatant types: B_1 and B_2, each with four elements. The B_1 and B_2 combatant types were assumed identical to evaluate the consistency between the estimated measures of WSE for the two. The engagement opened at an initial range of 2200 meters with the B force closing at a constant velocity of 12 kilometers per hour to a range of 200 meters, where the engagement was terminated. The following hypothetical attrition functions were used for the simulation:

$$a_{11}[\bar{r}(t),\bar{m},\bar{n}] = m_1\left(\frac{n_1}{n_1+n_2}\right)(.40),$$

$$a_{12}[\bar{r}(t),\bar{m},\bar{n}] = m_1\left(\frac{n_2}{n_1+n_2}\right)(.40),$$

$$a_{21}[\bar{r}(t),\bar{m},\bar{n}] = n_1(.50-.00020r),$$

$$a_{22}[\bar{r}(t),\bar{m},\bar{n}] = n_2(.50-.00020r),$$

$$b_{11}[\bar{r}(t),\bar{m},\bar{n}] = n_1 m_1(.22-.0001r),$$

$$b_{12}[\bar{r}(t),\bar{m},\bar{n}] = (.30-.0001r),$$

$$b_{21}[\overline{r}(t),\overline{m},\overline{n}] = n_2 m_1(.22-.0001r), \text{ and}$$

$$b_{22}[\overline{r}(t),\overline{m},\overline{n}] = (.30-.0001r),$$

where r is the range between combatants in meters.

Two independent sets of data were generated, the first with 20 replicates of the engagement and the second with 40 replicates to compare the analysis model under widely differing sample sizes. A total of 190 and 390 casualties were observed in the first and second simulations respectively and resulted in the LER shown in Table 1. With 20 repetitions of the engagement, LER indicated that A_2 was more than twice as effective against B_1 as against B_2. With 40 repetitions of the engagement, LER indicated A_1 was nearly twice as effective against B_2 as against B_1. In both cases, the hypothesized effectiveness against the two combatant types were identical and the anomalies in the LER were the result of the chance occurrence of disproportionate casualties during a part of the engagement.

3. Analysis

Figures three through eight compare the average values of the actual and estimated attrition functions for the observed time sequences of casualties in the two simulations. The estimated values of WSE agree closely with the actual values and generally demonstrate similar behavior over the course of the battle. Figure nine compares the average estimated values of WSE for A_2 against B_1 and B_2 and A_1 against B_1 and B_2 and shows that, as expected and unlike the LER, the estimated capability of each weapon type is nearly the same against B_1 as against B_2.

Table two compares the hypothetical and estimated attrition functions for the two simulations. The correlation nature of the Markov model is apparent in that, despite the lack of significant differences in the

Simulation Loss Exchange Ratios

time interval (minutes)	loss exchange ratio							
	20 repetitions				40 repetitions			
	A_1/B_1	A_1/B_2	A_2/B_1	A_2/B_2	A_1/B_1	A_1/B_2	A_2/B_1	A_2/B_2
0-1	5.0	2.5	--[1]	2.5	--[1]	3.5	5.0	2.6
0-2	5.3	2.0	13.0	6.0	6.7	2.9	6.4	3.7
0-3	3.7	3.2	12.0	5.6	2.1	2.8	4.5	2.9
0-4	3.0	2.7	10.3	5.0	2.2	2.8	4.8	3.2
0-5	3.1	3.4	8.8	4.0	1.9	3.1	4.4	3.0
0-6	3.1	2.9	10.5	3.8	1.7	3.2	4.3	3.3
0-7	3.1	2.6	10.8	3.8	1.8	3.1	4.4	3.4
0-8	3.1	2.6	10.8	4.0	1.6	3.2	4.3	3.4
0-9	3.1	2.6	8.8	3.7	1.6	3.1	4.1	3.3

[1] LER undefined

Table I

Comparison of Average Observed Values of Actual and Estimated Attrition Functions, 20 Trials

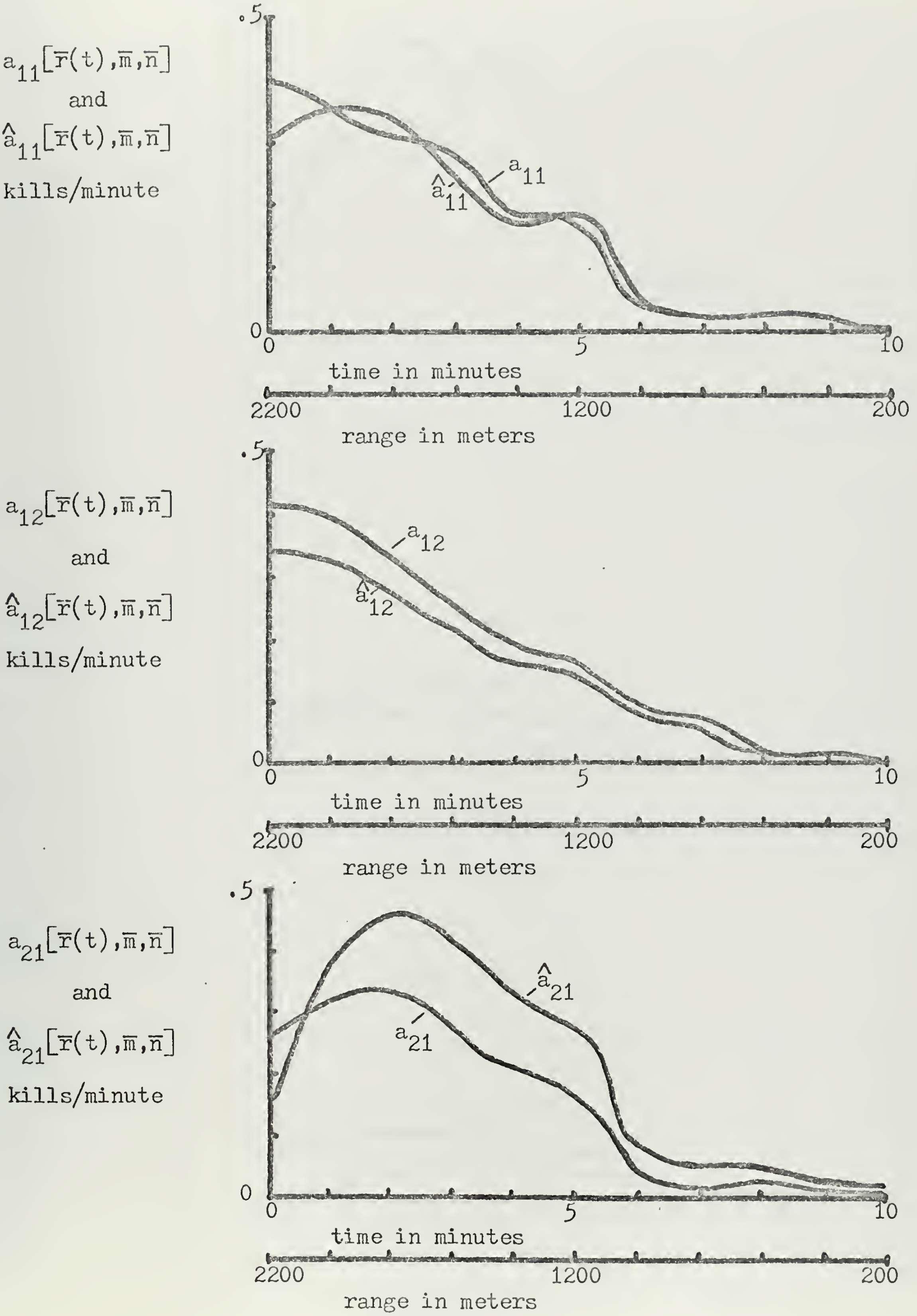


Figure 3

Comparison of Average Observed Values of Actual and Estimated Attrition Functions, 20 Trials

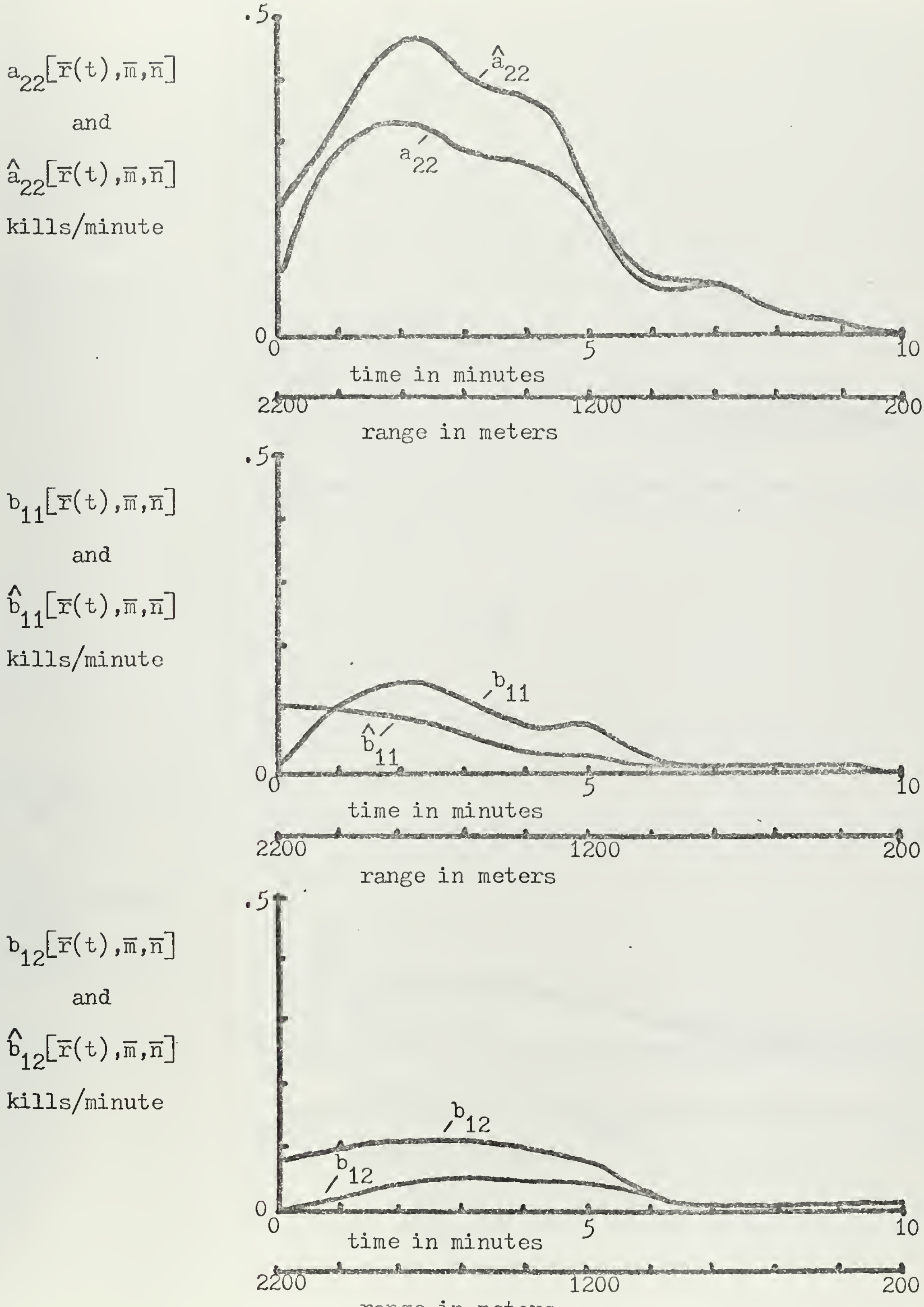


Figure 4

Comparison of Average Observed Values of Actual and Estimated Attrition Functions, 20 Trials

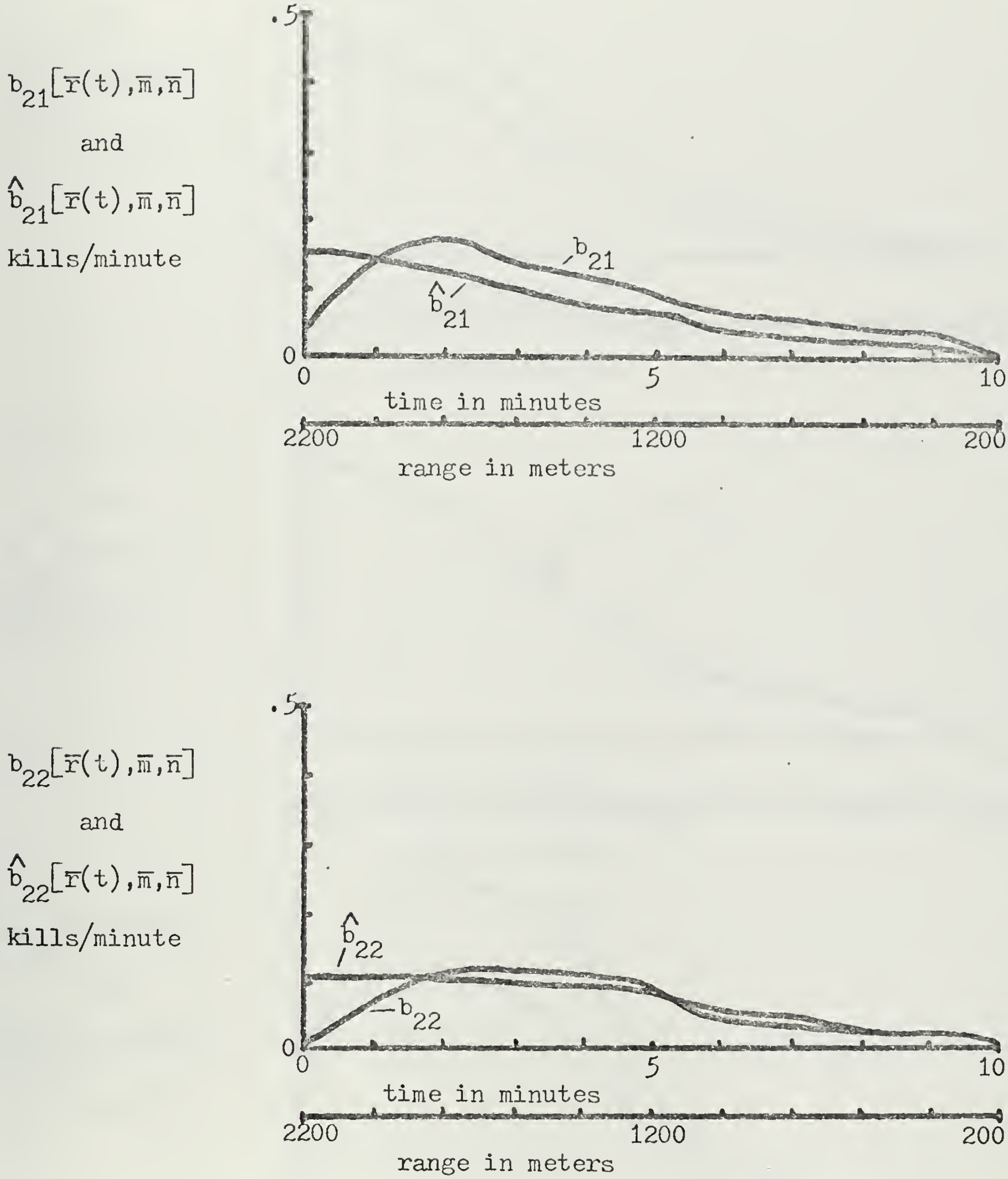


Figure 5

Comparison of Average Observed Values of Actual and Estimated Attrition Functions, 40 Trials

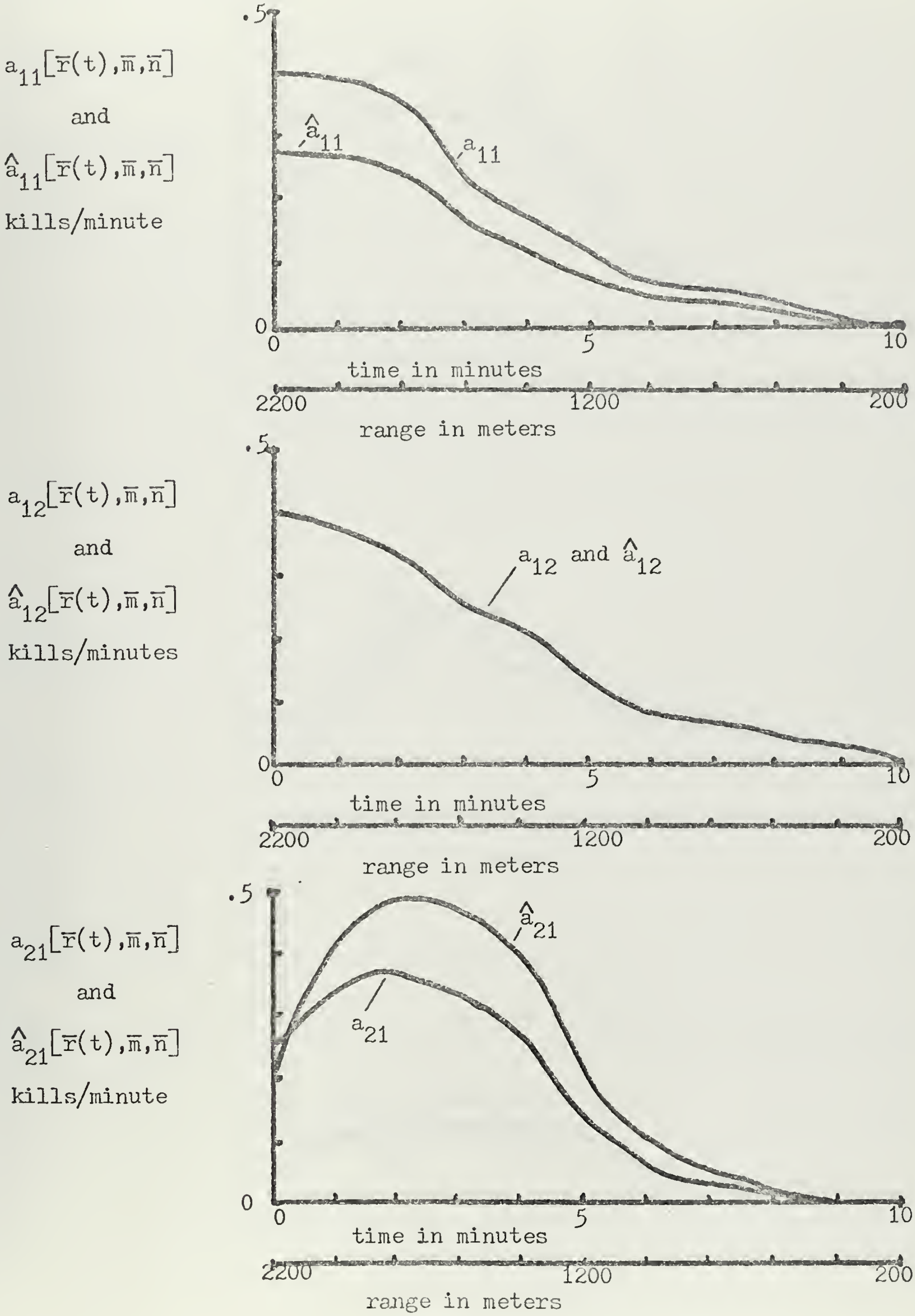


Figure 6

Comparison of Average Observed Values of Actual and Estimated Attrition Function, 40 Trials

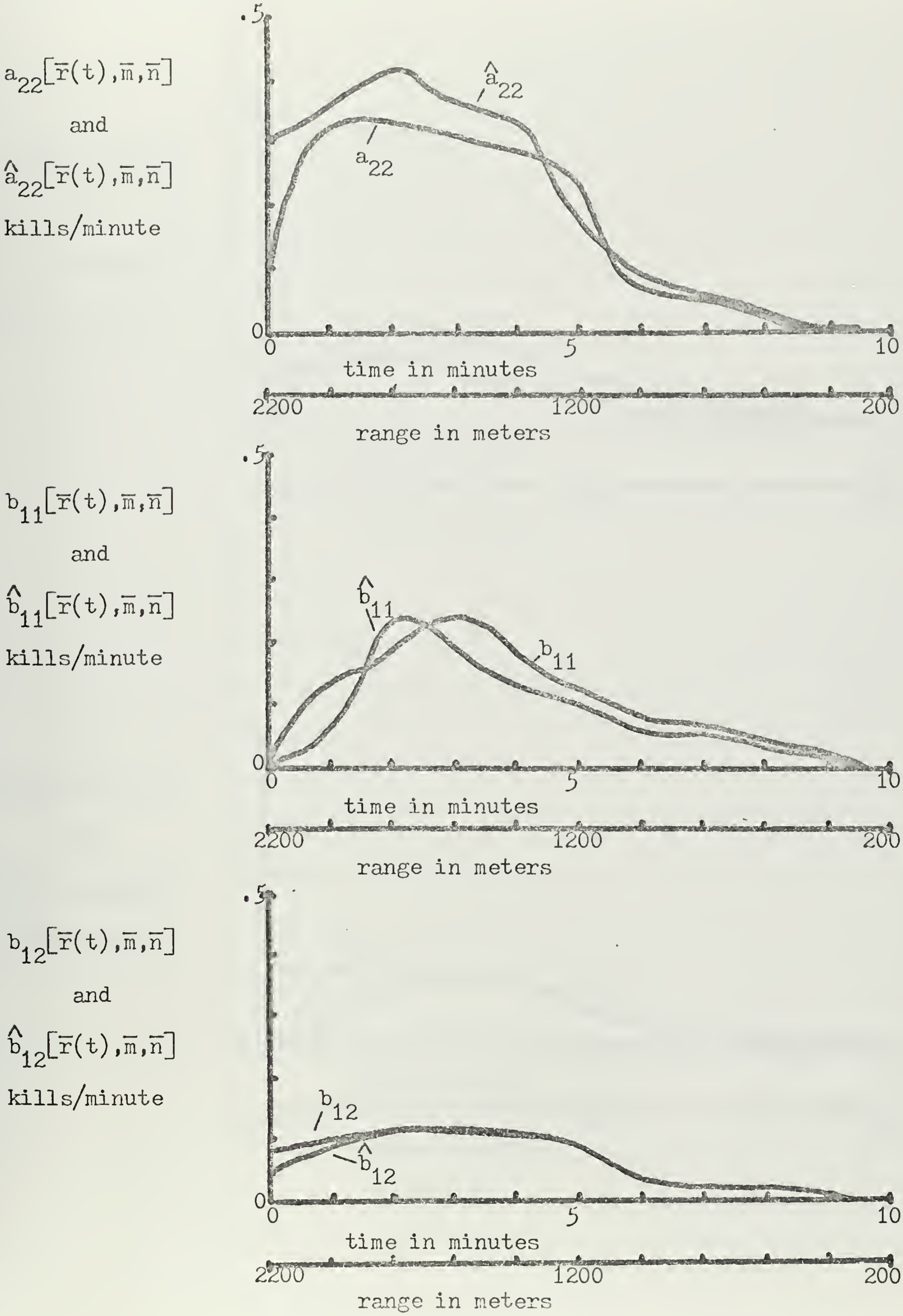
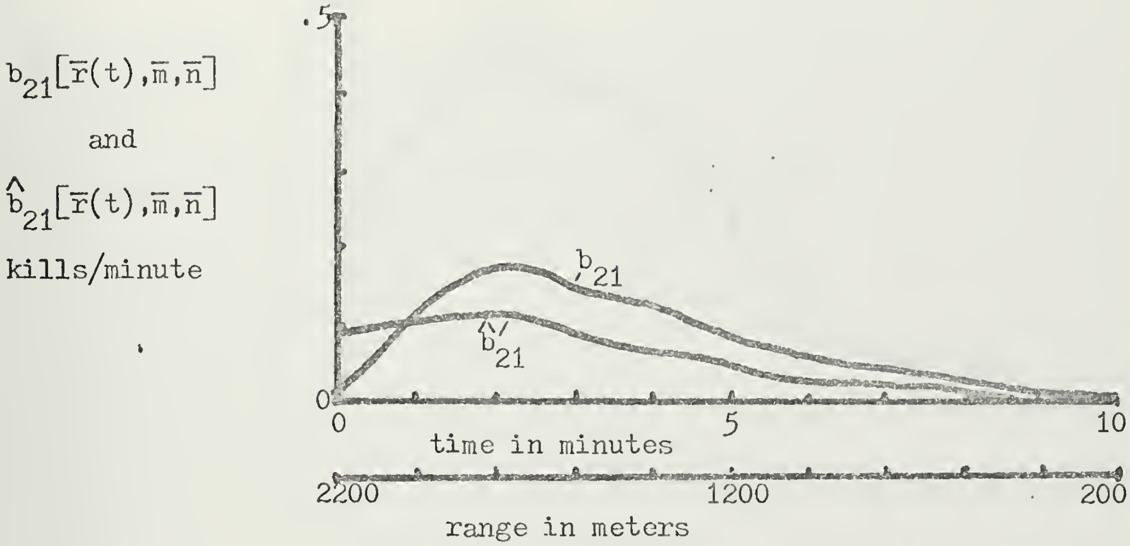


Figure 7

Comparison of Average Observed Values of Actual and Estimated Attrition Functions, 40 Trials



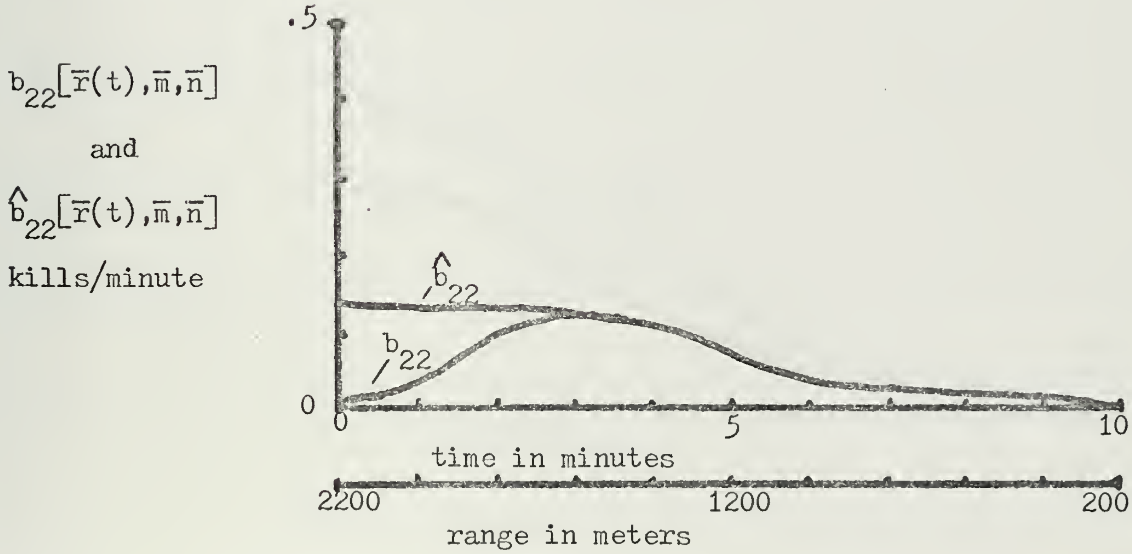


Figure 8

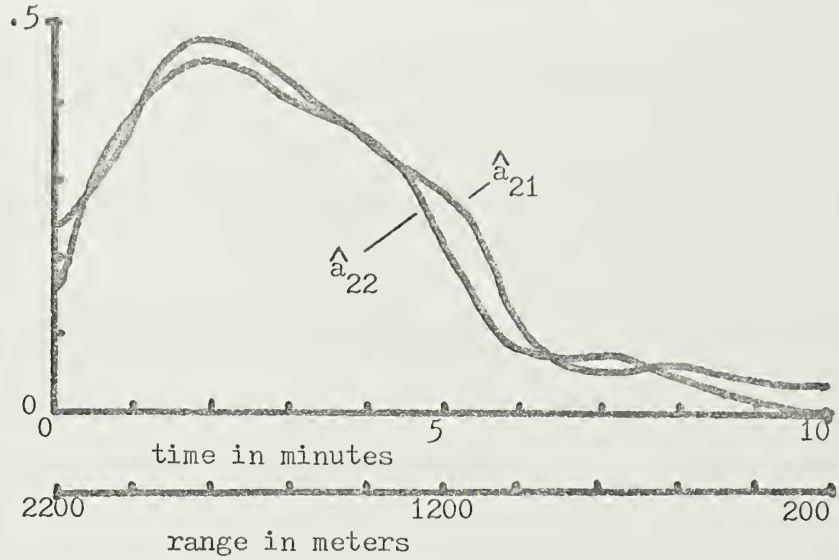
Comparison of A_2 Effectiveness Against B_1 and B_2, Average Observed Values of Estimated Attrition Functions, 20 Trials.

$\hat{a}_{21}[\bar{r}(t),\bar{m},\bar{n}]$

and

$\hat{a}_{22}[\bar{r}(t),\bar{m},\bar{n}]$

kills/minute



Comparison of A_1 Effectiveness Against B_1 and B_2, Average Observed Values of Estimated Attrition Functions, 40 Trials.

$\hat{a}_{11}[\bar{r}(t),\bar{m},\bar{n}]$

and

$\hat{a}_{12}[\bar{r}(t),\bar{m},\bar{n}]$

kills/minute

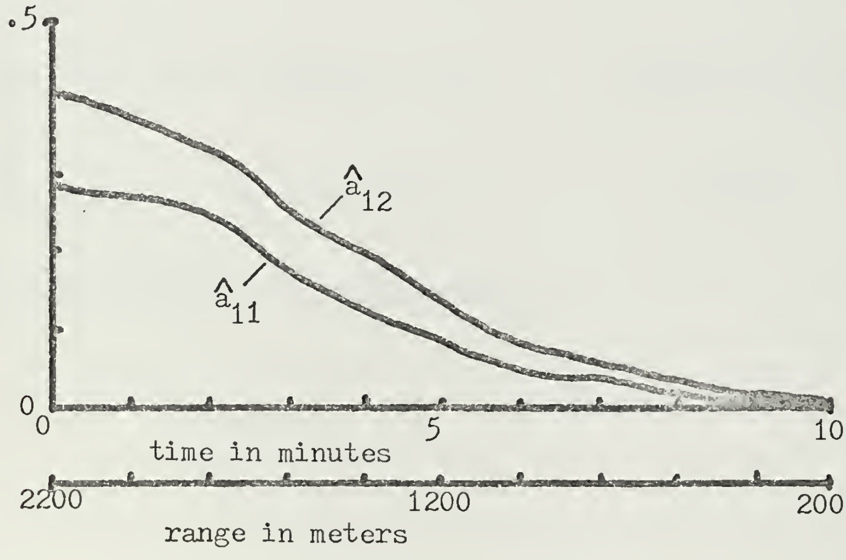


Figure 9

Comparison of Actual and Estimated Attrition Functions

attrition function	actual attrition function	estimated attrition functions 20 trials	40 trials
$a_{11}[\bar{r}(t),\bar{m},\bar{n}]$	$m_1(\frac{n_1}{n_1+n_2})(.40)$	$m_1 n_1(\frac{n_1}{n_1+n_2})(.578-.00022r)$	$m_1(\frac{n_1}{n_1+n_2})(.280)$
$a_{12}[\bar{r}(t),\bar{m},\bar{n}]$	$m_1(\frac{n_2}{n_1+n_2})(.40)$	$m_1(\frac{n_2}{n_1+n_2})(.332)$	$m_1(\frac{n_2}{n_1+n_2})(.398)$
$a_{21}[\bar{r}(t),\bar{m},\bar{n}]$	$n_1(.50-.00020r)$	$n_1(.914-.00040r)$	$n_1(.821-.00035r)$
$a_{22}[\bar{r}(t),\bar{m},\bar{n}]$	$n_2(.50-.00020r)$	$m_2 n_2(.47-.00020r)$	$n_2(.589-.00023r)$
$b_{11}[\bar{r}(t),\bar{m},\bar{n}]$	$n_1 m_1(.22-.0001r)$	$m_1(.060)$	$n_1 m_1(.548-.00047r)$
$b_{12}[\bar{r}(t),\bar{m},\bar{n}]$	$(.30-.0001r)$	$m_2(.132-.00005r)$	$m_2(.274-.00011r)$
$b_{21}[\bar{r}(t),\bar{m},\bar{n}]$	$n_2 m_1(.22-.0001r)$	$m_1(\frac{m_1}{m_1+m_2})(.151)$	$n_2 m_1(.097-.00003r)$
$b_{22}[\bar{r}(t),\bar{m},\bar{n}]$	$(.30-.0001r)$	$(\frac{m_2}{m_1+m_2})(.196)$	$(.132)$

r is the range in meters between opposing combatants

Table II

agreement of actual and estimated values of WSE between the two experiments, only the analysis based on forty repetitions was able to closely reproduce the attrition function forms and parameters used in the simulations.

Given the validity of the Markov assumptions, this analysis indicated that even for relatively small sample sizes- approximately half of the sample size of CDEC experiment 43.6- the model yields satisfactory estimates of the values of WSE demonstrated over the course of an engagement. However, considerably larger sample sizes are needed to develop a causative rather than correlation model of the attrition process for predictive purposes.

Appendix B. Analysis of Field Experimentation Data

1. General

This appendix presents the results of an analysis of CDEC experiment 43.6 using the Markov model. Information and data on the experiment were extracted from the unclassified Experimentation Plan and Volumes I and II of the Final Report, USACDEC Experiment 43.6, Attack Helicopter-Daylight Defense, ACN 18171.

2. Experiment 43.6

CDEC experiment 43.6, phase II, was a field experiment simulating a two-sided engagement between an attacking tank heavy force with supporting antihelicopter weapons and a defending force supported by antitank-missile firing helicopters. A primary purpose of the experiment was to evaluate the contribution of the missile firing helicopters to the capability of the defending force. The nominal initial composition of each force is shown below:

Defender, A force			Attacker, B force		
combatant type	description	number elements	combatant type	description	number elements
A_1	COBRA helicopter with TOW	2	B_1	Tank	8
A_2	Observation helicopter	1	B_2	REDEYE, SAM on APC	1
A_3	TOW on APC	2	B_3	APC	2
A_4	Tank	2	B_4	23MM SPAA	1
			B_5	57MM SPAA	1

The engagement commenced at a range of approximately 3750 meters with the attaching force closing with the stationary defending force at a velocity of about ten kilometers per hour.

3. Data

The unclassified data available from experiment 43.6 consisted of casualty time histories from 46 repetitions of the above engagement performed in four similar areas. Data available from each experiment consisted of the initial force composition, the time and range at which each casualty occurred, and the type of and type unit inflicting each casualty. Since this analysis was intended as an exercise of the Markov model rather than a study of experiment 43.6, certain trials of the experiment were arbitrarily omitted from the analysis. Specifically, 15 trials, trial numbers 033, 184, 194, 203, 214, 233, 254, 263, 011, 132, 161, 072, 082, 171, and 201, were omitted due to instrumentation failure during the trial and/or the need for classified information to analyse the trial. The majority of the omitted trials could be included in the Markov analysis, but the purpose of this analysis did not warrent the effort. The omission of this data may have significantly biased the estimates of WSE, in that these trials incidentally included the ones most favorable to the attacking force. Again due to the limited objectives of this analysis, the data from different areas was pooled without regard to the effect of changes in terrain.

One type of instrumentation failure which occurred occasionally in experiment 43.6 was the failure to render an assessed casualty inoperative which allowed this combatant to continue participating in the engagement. The Markov model can deal with such a failure directly due to the assumption of the independence of non-overlapping time intervals. In the

analysis of a trial containing such a failure, the casualty is assessed as usual, but the vectors of surviving combatants used in the analysis of the next casualty are left unchanged.

Data casualty times were in clock, rather than lapsed time and, hence, the relationship between time and range was not directly comparable between trials. The relationship between time and range was approximated by assuming the attacking force closed at a constant velocity of 10 kilometers per hour and least square criteria was used to convert casualty times into lapsed times from the beginning of each trial, where $r(t=0)=$ 3750 meters. Table 3 lists the adjusted time for the occurrence of the first casualty in each trial.

4. Analysis

The CDEC references on the experiment indicated that the combatants, except for A_1 and B_2, were ineffective beyond a range of 2500 meters. However, the data records kills by other combatant types at ranges in excess of 2800 meters. The analysis is based on the assumption that all combatants, except A_1 and B_2, are ineffective at ranges beyond 2900 meters. Figures 10, 11, and 12A plot the average values of estimated WSE for each combatant against range and time for the observed time sequences of casualties in the 31 trials. The data base did not include any casualties inflicted by combatants A_2, B_3, B_4, and B_5 nor any casualties inflicted by A_1 on B_2 or B_1 on A_2. Figure 12B indicates the contribution of combatant A_1 (COBRA) to the effectiveness of the A force against combatant B_1 (TANK). Tables four and five list the estimated attrition functions for the combatants.

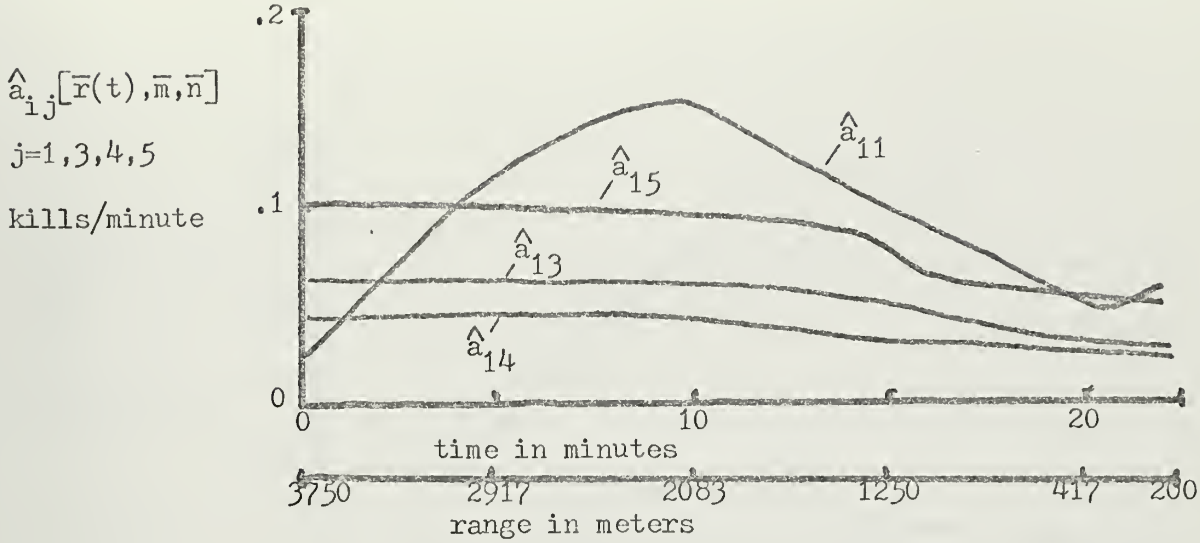
Adjusted Time of Occurrence of First Casualty

Times given in minutes

experiment	t_1	experiment	t_1
023	7.482	382	4.846
123	9.696	102	2.692
144	9.602	092	4.404
154	5.646	111	11.112
173	5.434	021	8.538
223	0.944	031	10.256
244	5.996	051	10.632
083	3.428	062	0.924
273	9.294	121	10.608
293	11.394	142	3.620
304	4.100	282	5.672
314	12.070	212	4.322
323	3.450	221	10.618
333	3.218	242	3.288
344	5.354	261	10.154
363	3.344		

Table III

Average Observed Effectiveness of Combatant A_1 (COBRA)



Average Observed Effectiveness of Combatant A_3 (TOW)

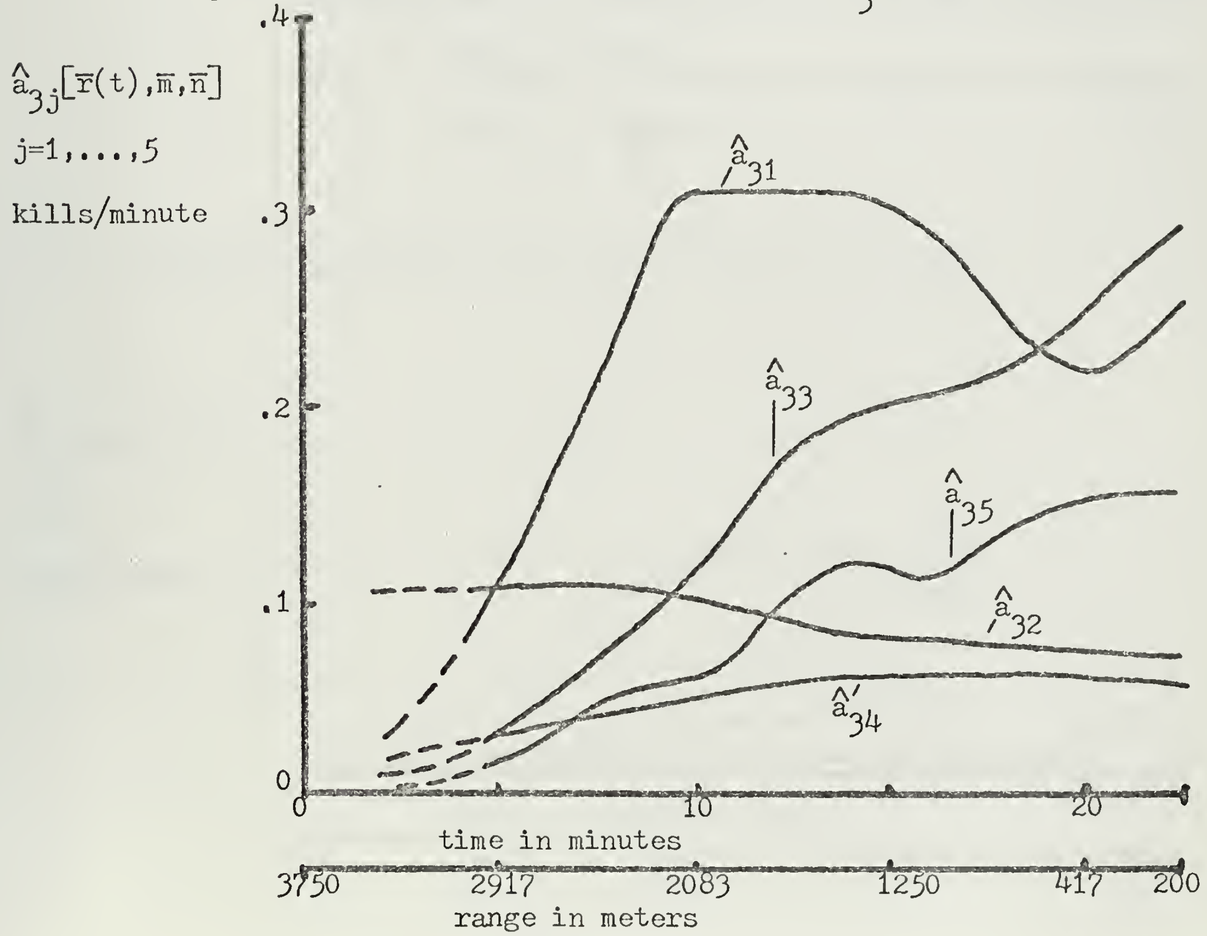
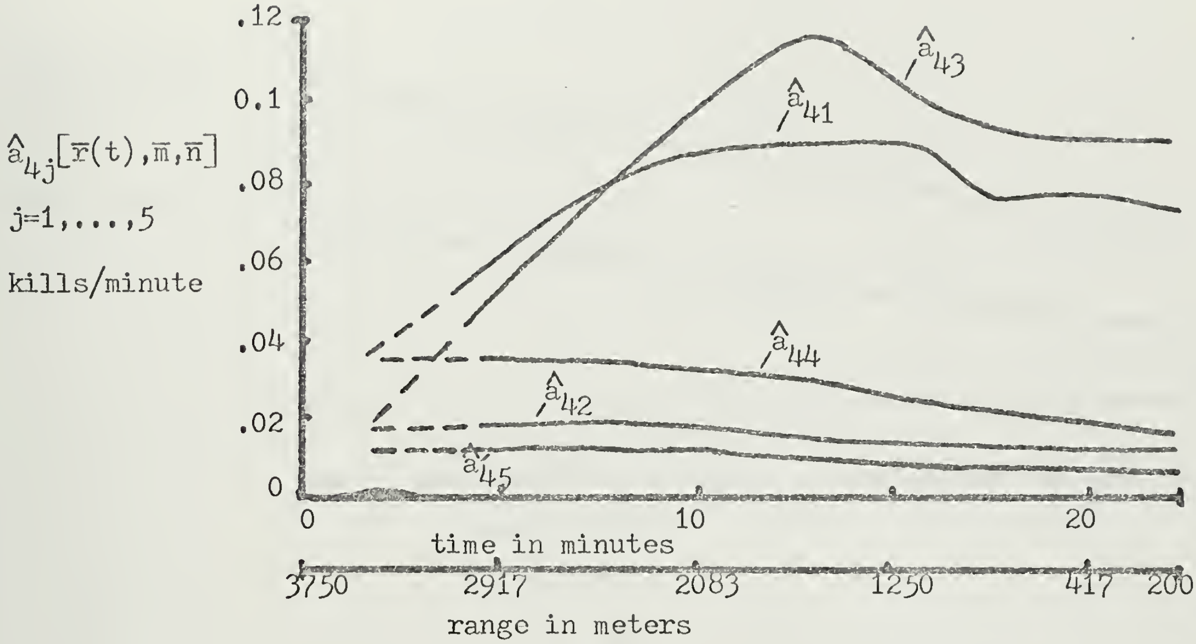


Figure 10

Average Observed Effectiveness of Combatant A_4 (TANK)



Average Observed Effectiveness of Combatant B_1 (TANK)

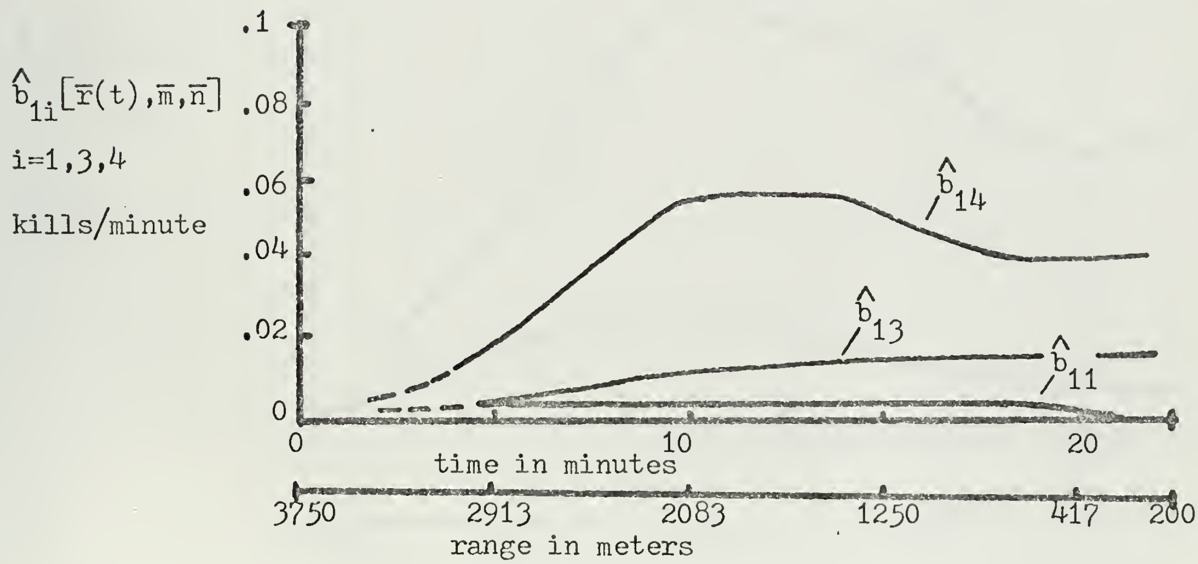


Figure 11

Average Observed Effectiveness of Combatant B_2 (REDEYE)

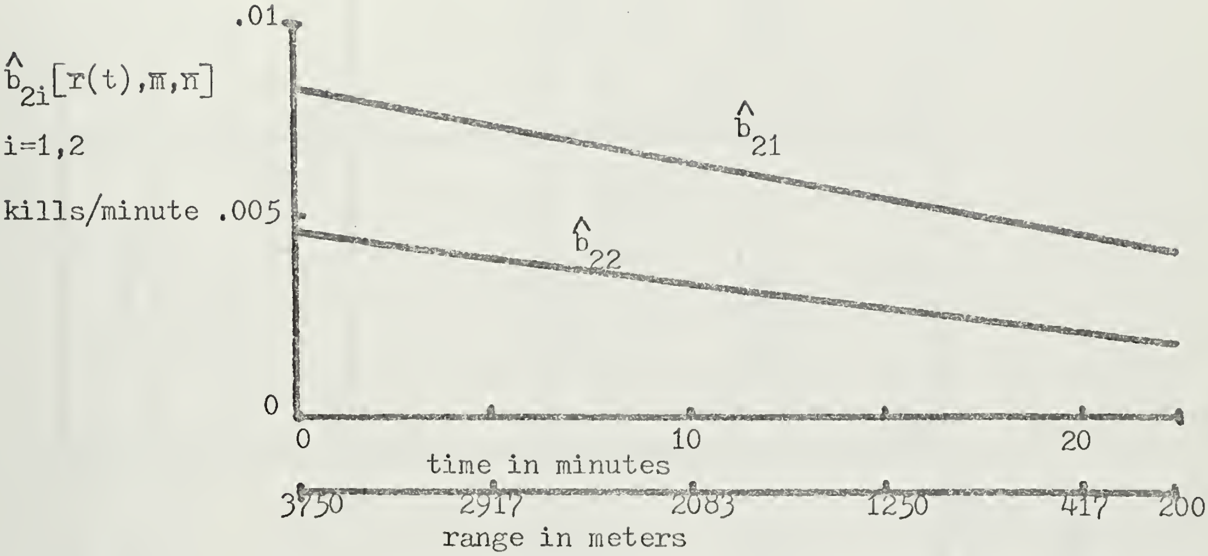


Figure 12A

Contribution of Combatant A_1 (COBRA) to Effectiveness of A Force Against Combatant B_1 (TANK)

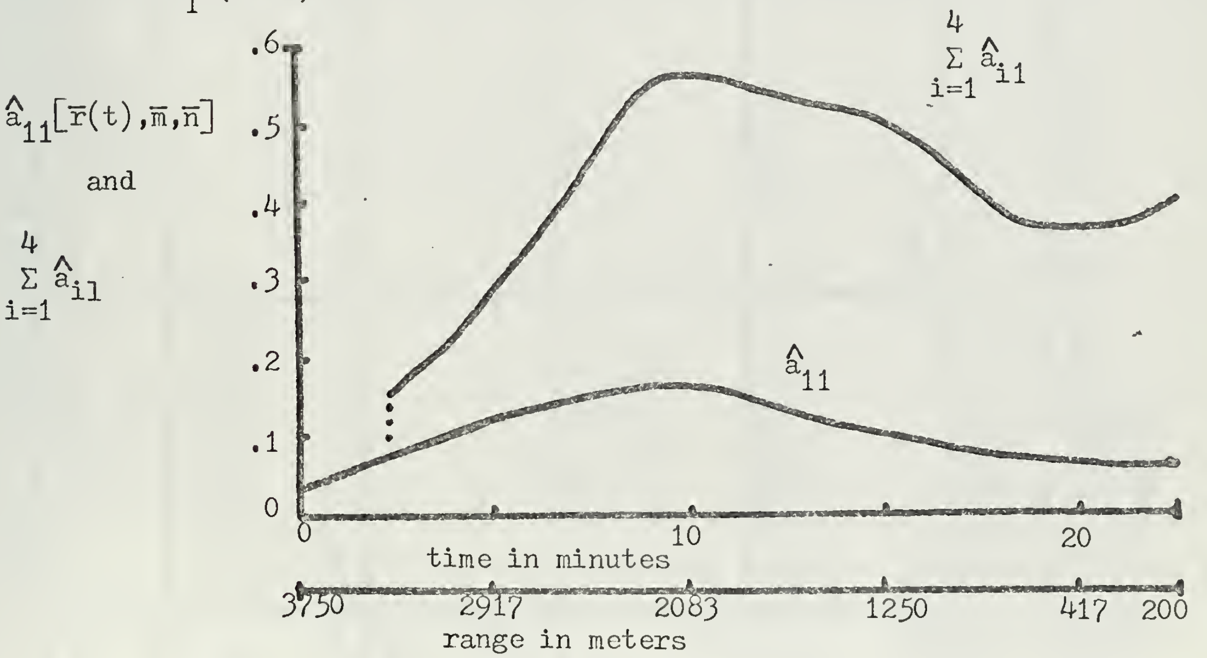


Figure 12B

Estimated Attrition Functions for Side A, $\hat{a}_{ij}[\bar{r}(t),\bar{m},\bar{n}]$

i \ j	B_1 TANK n_1 elements	B_2 REDEYE n_2 elements	B_3 APC n_3 elements	B_4 23 MM n_4 elements	B_5 57 MM n_5 elements
A_1 COBRA m_1 elements	geometric[1] $n_1A_1(.10-.000026r)$ 299[2]	——— [3]	square $m_1A_3(.063)$ 260	square $m_1A_4(.044)$ 240	constant $A_5(.10)$ 245
A_2 SCOUT m_2 elements	——— [4]	——— [4]	——— [4]	——— [4]	——— [4]
A_3 TOW m_3 elements	geometric $n_1A_1(.46-.00024r +3.2X10^{-8}r^2)$ 288	square $m_3A_2(.11)$ 255	geometric $n_3A_3(.61-.00032r)$ 250	square $m_3A_4(.12-.000034r +4.3X10^{-8}r^2)$ 229	square $m_3A_5(.39-.00021r +2.7X10^{-8}r^2)$ 235
A_4 TANK m_4 elements	constant $A_1(.31-.000070r)$ 272	constant $A_2(.019)$ 242	constant $A_3(.24-.000060r)$ 237	square $m_4A_4(.036)$ 234	square $m_4A_5(.013)$ 222

$A_i = n_i/(n_1+n_2+n_3+n_4+n_5)$ $\qquad$ r = range in meters

1. Type dependence on number surviving combatants.
2. Number of observations used in estimating attrition function.
3. No kills observed.
4. A_2 had no armament.

Table IV

Estimated Attrition Functions for Side B, $\hat{b}_{ji}[\bar{r}(t),\bar{m},\bar{n}]$

j \ i	A_1 COBRA m_1 elements	A_2 SCOUT m_2 elements	A_3 TOW m_3 elements	A_4 TANK m_4 elements
B_1 TANK n_1 elements	geometric[1] $m_1B_1(.0064)$ 290[2]	——— [3]	constant $B_3(.072-.000020r)$ 288	square $n_1B_4(.15-.000080r)$ 272
B_2 REDEYE n_2 elements	geometric $m_1(.00384)$ 266	constant $(.00387)$ 254	——— [4]	——— [4]
B_3 APC n_3 elements	——— [5]	——— [5]	——— [5]	——— [5]
B_4 23 MM n_4 elements	——— [3]	——— [3]	——— [3]	——— [3]
B_5 57 MM n_5 elements	——— [3]	——— [3]	——— [3]	——— [3]

$B_i = m_i/(m_1+m_3+m_4)$ $\qquad$ r= range in meters

1. Type dependence on number surviving combatants.
2. Number of observations used in estimating attrition functions.
3. No kills observed.
4. B_2 ineffective against target types A_3 and A_4.
5. B_3 had no armament.

Table V

All A combatant types appeared to distribute their fire uniformly, but the B_1 combatants seemed to distribute their fire only over the A_1, A_3, and A_4 combatants without engaging the A_2 combatants. The geometric term or dependence on the number of targets which appears in attrition functions a_{11}, a_{31}, a_{33}, b_{11}, and b_{21} indicates a high dependence of WSE on the target acquisition process. Attrition function b_{22} might be expected to show a similar dependence had the data base included more than one A_2 (SCOUT) combatant. This result is consistent with the low altitude, pop-up tactics employed by the helicopters and the use of covered and/or concealed routes of approach by combatants B_1 (TANKS) and B_3 (APC). The fact that this dependence did not appear in the other attrition functions may have resulted from the relatively fixed positions of combatants A_3 (TOW) and A_4 (TANK) and a tendency for B_2 (REDEYE), B_4 (23 MM), and B_5 (57 MM) to use more exposed positions in order to engage the helicopters. However, note that the data is insufficient to support such conclusions about the attrition of B_2, B_4, and B_5 since there is only one element of each of these combatant types. Similarly, it is impossible to determine the relationship between B_2 (REDEYE) effectiveness and the number of B_2 elements with a data base of one B_2 combatant. Finally, due to the very small number of A_1 (COBRA) losses observed, it is doubtful whether the model would detect any dependence of A_1 WSE on the number of COBRA's, that is $a_{11}[\bar{r}(t),\bar{m},\bar{n}]$ might be $m_1 n_1 A_1(.05-.00013r)$ rather than $n_1 A_1(.10-.00026r)$. For the most part, the attrition functions in Tables four and five probably represent correlation rather than causative models of the attrition process and their use for predictive studies might prove risky.

The plots of average observed WSE all tend to display a decrease in effectiveness toward the end of the engagement attributable to the diminishing number of surviving firers and/or to the increasing scarcity of

targets. Some of the plots show WSE to increase during the earlier part of the battle which is probably due to increasing weapon accuracy at shorter ranges and/or the increasing ease of acquiring the remaining targets. This latter factor probably holds throughout the engagement, but is often outweighed by the diminishing numbers of firing and target combatants late in the engagement.

This discussion reveals some aspects of the comprehensive nature of the estimate of observed effectiveness of a combatant. This measure of WSE reflects not only the physical performance characteristics of a combatant type, but also decreases in WSE due to the system's vulnerability to opposing weapons and consequent losses as the battle proceeds. Moreover, the model reflects actual rather than potential WSE and may show WSE to decrease as the number of opposing combatants decrease due, say, to increasing difficulty in target acquisition and, of course, a system's WSE would go to zero when the opposing force was annihilated.

Appendix C

1. General

This appendix presents an attempt to develop a reasonably inclusive selection of potential general forms of the attrition function to consider in conducting sequential tests of hypothesis. The selection was designed to be sufficiently comprehensive to include known or suspected relationships between WSE and the attrition process, but limited enough to facilitate creating a standard program for the analysis of experimental data using the Markov model and sequential tests of hypothesis to select among the potential general forms of the attrition process. In addition, the consequent maximum likelihood estimators or simultaneous nonlinear equations which yield the estimators are indicated.

2. General Forms of the Attrition Function

Consideration was limited to attrition functions with the general form of a product of a force level dependent term, $f(\overline{m},\overline{n})$; a term representing a combatant type's allocation of effort over potential target types, $g(\overline{m},\overline{n})$; and a range dependent term, $h[\overline{r}(t)]$:

$$a_{ij}[\overline{r}(t),\overline{m},\overline{n}]=f(\overline{m},\overline{n}) \times g(\overline{m},\overline{n}) \times h[\overline{r}(t)].$$

Potential forms of the first or force level dependent term were selected by analogy to existing Lanchester combat models. Forms considered were:

$$a_{ij}[\overline{r}(t),\overline{m},\overline{n}]=1 \times g(\overline{m},\overline{n}) \times h[\overline{r}(t)],$$

$$a_{ij}[\overline{r}(t),\overline{m},\overline{n}]=m_i \times g(\overline{m},\overline{n}) \times h[\overline{r}(t)],$$

$$a_{ij}[\overline{r}(t),\overline{m},\overline{n}]=n_j \times g(\overline{m},\overline{n}) \times h[\overline{r}(t)], \text{ and}$$

$$a_{ij}[\overline{r}(t),\overline{m},\overline{n}]=m_i n_j \times g(\overline{m},\overline{n}) \times h[\overline{r}(t)].$$

Again by analogy to Lanchester theory, these forms of the attrition functions are referred to as constant, square law, geometric law, and linear law forms respectively.

The allocation of effort term was assumed to be either constant or proportional to the relative numbers of the opposing combatants. The constant allocation term simply represents the allocation of a constant amount of firer effort to each target type. The second form may be interpreted as the allocation of effort in proportion to the relative numbers of each type of opposing combatant or as the result of a target acquisition process where the probability of acquiring a given target type is proportional to the number of that target type. The attrition function may then be written as:

$$a_{ij}[\bar{r}(t),\bar{m},\bar{n}]=f(\bar{m},\bar{n}) \text{ X } 1 \text{ X } h[\bar{r}(t)] \quad \text{or}$$

$$a_{ij}[\bar{r}(t),\bar{m},\bar{n}]=f(\bar{m},\bar{n}) \text{ X } (w_j n_j / \sum_{k=1}^{d} w_k n_k) \text{ X } h[\bar{r}(t)],$$

where w_j, j=1, ..., d are unknown parameters to be estimated which represent the relative importance of each opposing combatant in the allocation of effort. The last or range dependent term was assumed to be a linear function of range: $h[\bar{r}(t)]= a+br+cr^2+....$, where a,b,c,... are unknown parameters to be estimated.

It was felt that the variety of attrition function forms resulting from the possible combinations of the terms presented above would prove computationally tractable and sufficiently general to describe most relationships between WSE and the state of the attrition process. Appendices A and B contain examples of analyses conducted using the above forms of the attrition functions with $h[\bar{r}(t)]$ constrained to be of quadratic or lower order in r and w_j in the allocation term one or zero, j=1,...,d.

3. Maximum Likelihood Estimators

The special form of the attrition function $a_{ij}[\bar{r}(t),\bar{m},\bar{n}]=m_i a$ is noteworthy in that the maximum likelihood estimator of the parameter a is given by:

$$\hat{a} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} x_{ijkl} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} m_{ikl}(t_{kl}-t_{(k-1)l}),$$

which is the number of B_j casualties inflicted by A_i combatants per time unit of A_i effort in inflicting these casualties.

In general, for $g(\bar{m},\bar{n})= 1$ and $h[\bar{r}(t)]= a$:

$$\hat{a} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} x_{ijkl} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}-t_{(k-1)l}).$$

For $g(\bar{m},\bar{n}) = 1$ and $h[\bar{r}(t)] = (a'-b'r) = (a+bt)$, $\hat{a}$ and $\hat{b}$ are given by the solutions to the simultaneous equations:

$$\hat{a} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \frac{a\, x_{ijkl}}{a+bt_{kl}} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}-t_{(k-1)l}), \text{ and}$$

$$\hat{b} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \frac{b\, x_{ijkl}\, t_{kl}}{a + bt_{kl}} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \tfrac{1}{2}f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}^2-t_{(k-1)l}^2).$$

For $g(\bar{m},\bar{n}) = 1$ and $h[\bar{r}(t)] = (a'-b'r-c'r^2) = (a+bt+ct^2)$; $\hat{a}$, $\hat{b}$, and $\hat{c}$ are given by the solutions to the simultaneous equations:

$$\hat{a} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \frac{a x_{ijkl}}{a+bt_{kl}+ct_{kl}^2} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}-t_{(k-1)l}),$$

$$\hat{b} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \frac{b x_{ijkl} t_{kl}}{a+bt_{kl}+ct_{kl}^2} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \tfrac{1}{2}f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}^2-t_{(k-1)l}^2), \text{ and}$$

$$\hat{c} = \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \frac{c x_{ijkl} t_{kl}^2}{a+bt_{kl}+ct_{kl}^2} \Big/ \sum_{l=1}^{L} \sum_{k=1}^{K(l)} \tfrac{1}{3}f(\bar{m}_{kl},\bar{n}_{kl})(t_{kl}^3-t_{(k-1)l}^3).$$

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A nonstationary Markov model was developed relating weapon system effectiveness (WSE) to the time sequence of casualties observed in a two-sided, heterogeneous force, land combat simulation or field experiment. Parametric tests of hypothesis are used to analyse the relationships between WSE and engagement range and the numbers and types of combatants in the engagement. Maximum likelihood estimators of WSE demonstrated by a weapon system over the course of an engagement are

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obtained. Utilization of the Markovian model and WSE estimates in a low resolution simulation to investigate the impact of changes in force mix, speed of advance, and initial engagement range is discussed.

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